

• FUNÇÃO QUADRÁTICA •

DEFINIÇÃO

DOMÍNIO

CONTRA
DOMÍNIO

Uma função $f: \mathbb{R} \rightarrow \mathbb{R}$ é dita quadrática quando existem constantes $a, b, c \in \mathbb{R}$, $a \neq 0$, tais que $f(x) = ax^2 + bx + c$, para todo $x \in \mathbb{R}$.

EXEMPLOS

$$\text{i) } f(x) = \overset{\textcolor{red}{a}}{\textcolor{red}{3}}x^2 + \overset{\textcolor{red}{b}}{\textcolor{red}{2}}x + \overset{\textcolor{red}{c}}{\textcolor{red}{5}}$$

$$\text{ii) } g(x) = \overset{\textcolor{red}{a}}{\textcolor{red}{-1}}x^2 + \overset{\textcolor{red}{b}}{\textcolor{red}{5}}x - \overset{\textcolor{red}{c}}{\textcolor{red}{8}}$$

$$\text{iii) } h(x) = \overset{\textcolor{red}{a}}{\textcolor{red}{-2}}x^2 + \overset{\textcolor{red}{c}}{\textcolor{red}{5}} \quad (b=0)$$



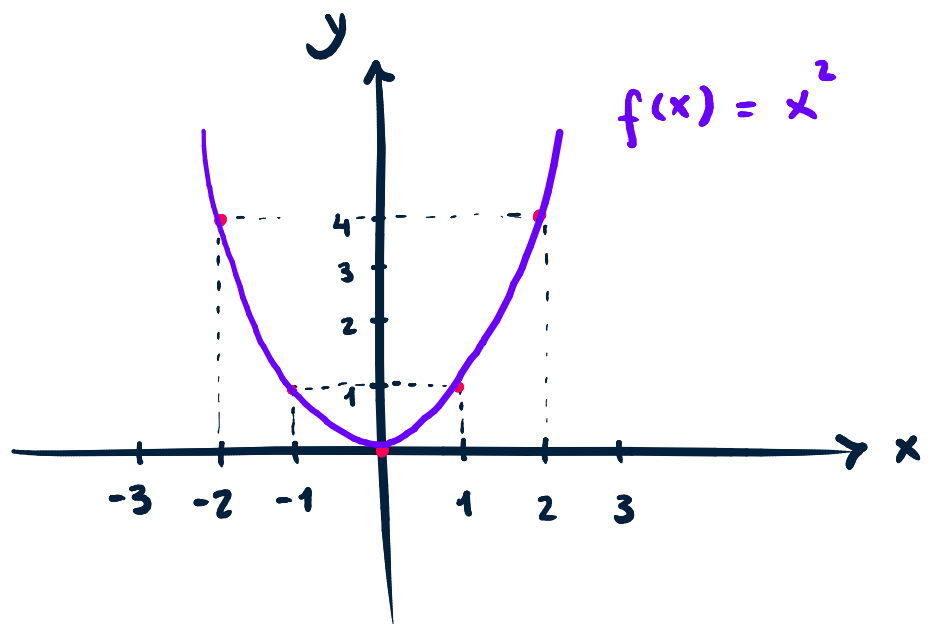
VISUALIZAÇÃO

= $\boxed{x}$

$$f(x) = x^2$$

= $\boxed{y}$

x	y
0	0
1	1
-1	1
2	4
-2	4

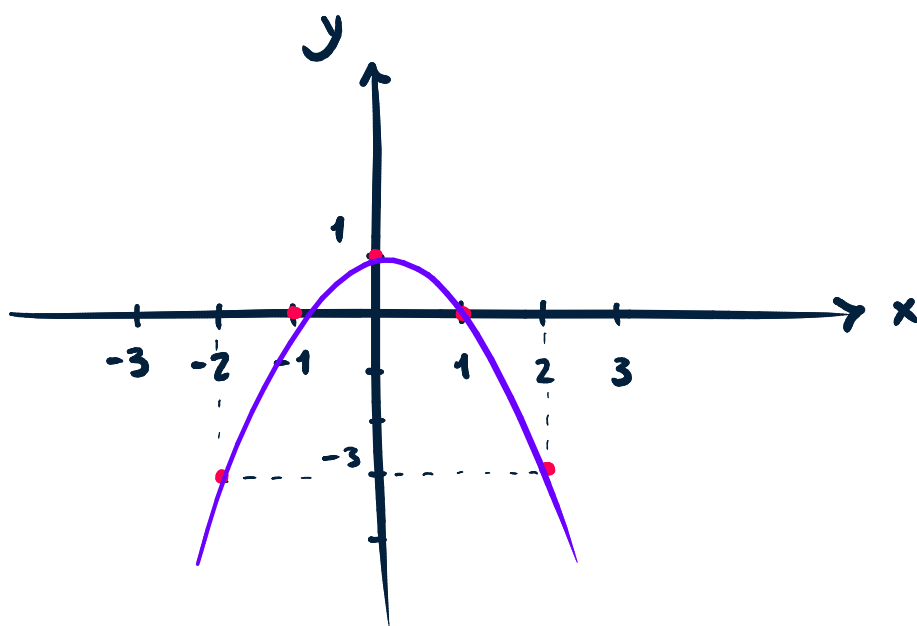


= x

$$f(x) = -x^2 + 1$$

= y

x	y
0	1
1	0
-1	0
2	-3
-2	-3



FORMA CANÔNICA

$$f(x) = ax^2 + bx + c = ax^2 + b \frac{a}{a}x + c \frac{a}{a}$$

$$= a \left(x^2 + \frac{b}{a}x + \frac{c}{a} \right)$$

$$= a \left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a} \right)^2 - \left(\frac{b}{2a} \right)^2 + \frac{c}{a} \right)$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 + \frac{4a \cdot c}{4a^2} - \frac{b^2}{4a^2} \right]$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{(b^2 - 4ac)}{4a^2} \right]$$

$$f(x) = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{(b^2 - 4ac)}{4a^2} \right]$$



FORMA CANÔNICA

↳ RAÍZES

$$\Delta = b^2 - 4ac$$

$$f(x) = ax^2 + bx + c = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{(b^2 - 4ac)}{4a^2} \right]$$

↳ RAÍZES: $f(x) = 0$

$$a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right] = 0$$

$$\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} = 0 \quad \therefore \left(x + \frac{b}{2a} \right)^2 = \frac{\Delta}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{\Delta}}{2a} \quad \therefore x = -\frac{b}{2a} \pm \frac{\sqrt{\Delta}}{2a}$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\rightarrow x_1 : \frac{-b + \sqrt{\Delta}}{2a}$$

$$\rightarrow x_2 : \frac{-b - \sqrt{\Delta}}{2a}$$



FORMA CANÔNICA

↳ RAÍZES

• EXEMPLO 01 •

Encontre as raízes de $f(x) = x^2 + 10x + 25$

R:

i) APLICAÇÃO DA FÓRMULA

$$\Delta = 10^2 - 4 \cdot 1 \cdot 25 = 0$$

$$\hookrightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-10 \pm \sqrt{0}}{2} = -\frac{10}{2} = -5 \quad \downarrow$$

ii) COMPLETANDO QUADRADOS

$$f(x) = x^2 + 10x + 25 = 0$$

$$x^2 + 2 \cdot x \cdot 5 + 5^2 = 0$$

$$(x + 5)^2 = 0 \quad \therefore \boxed{x = -5}$$



FORMA CANÔNICA

↳ RAÍZES

• EXEMPLO 02 •

Encontre as raízes de $f(x) = x^2 - 8x + 1$

$$a = 1$$

$$b = -8$$

$$c = 1$$

R :

i) APLICAÇÃO DA FÓRMULA

$$\Delta = (-8)^2 - 4 \cdot 1 \cdot 1 = 64 - 4 = 60$$

$$x = \frac{8 \pm \sqrt{60}}{2 \cdot 1} \quad \begin{array}{l} \nearrow x_1 = \frac{8 + \sqrt{60}}{2} = \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15} \\ \searrow x_2 = \frac{8 - \sqrt{60}}{2} = \frac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15} \end{array}$$

ii) COMPLETANDO QUADRADOS

$$f(x) = x^2 - 8x + 1 = 0 \quad \therefore x^2 - 8x + 1 + 15 = 0 + 15$$

$$x^2 - 8x + 16 = 15 \quad \therefore (x - 4)^2 = 15$$

$$x - 4 = \pm \sqrt{15} \quad \therefore x = 4 \pm \sqrt{15}$$

$$\nearrow x_1 = 4 + \sqrt{15}$$

$$\searrow x_2 = 4 - \sqrt{15}$$

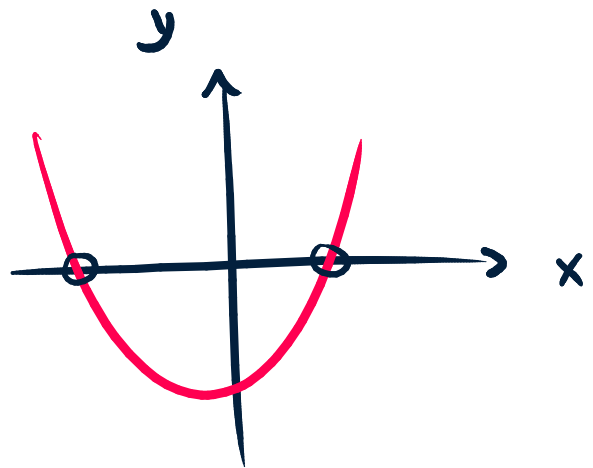


FORMA CANÔNICA

↳ RAÍZES : $x = \frac{-b \pm \sqrt{\Delta}}{2a}$

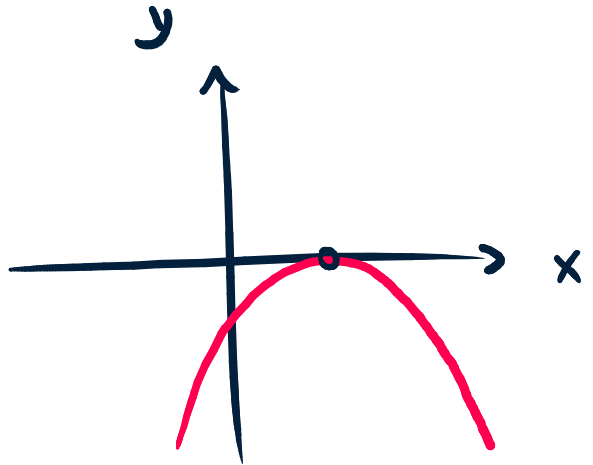
CASO 01 : $\Delta > 0$

2 RAÍZES DISTÍNTAS



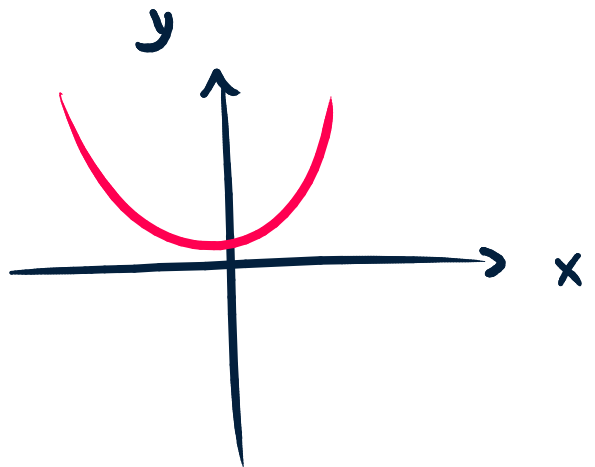
CASO 02 : $\Delta = 0$

2 RAÍZES IGUAIS.



CASO 03 : $\Delta < 0$

NÃO HÁ NENHUMA
RAÍZ REAL



FORMA CANÔNICA

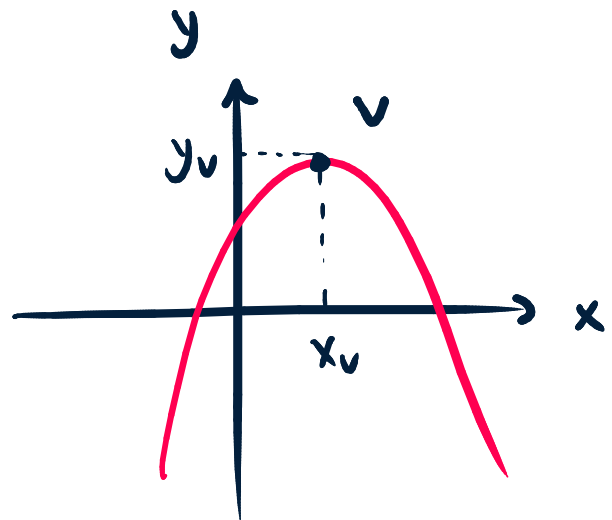
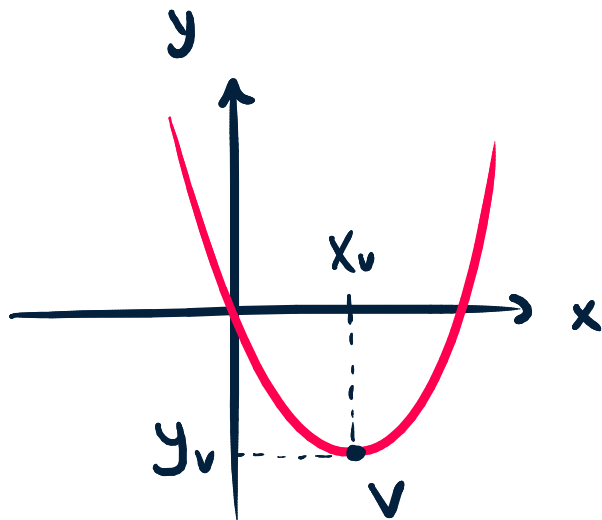
↳ MÁXIMO E MÍNIMO

$$\Delta = b^2 - 4ac$$

$$f(x) = ax^2 + bx + c = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right]$$

Máximo/Mínimo : $\left(x + \frac{b}{2a} \right)^2 = 0 \therefore \boxed{x_v = -\frac{b}{2a}}$

$\rightarrow y_v = f(x_v) = -\frac{\Delta}{4a} \therefore \boxed{y_v = \frac{-\Delta}{4a}}$



Coordenadas do Vértice : $V = (x_v ; y_v)$

$$= \left(-\frac{b}{2a} ; \frac{-\Delta}{4a} \right)$$



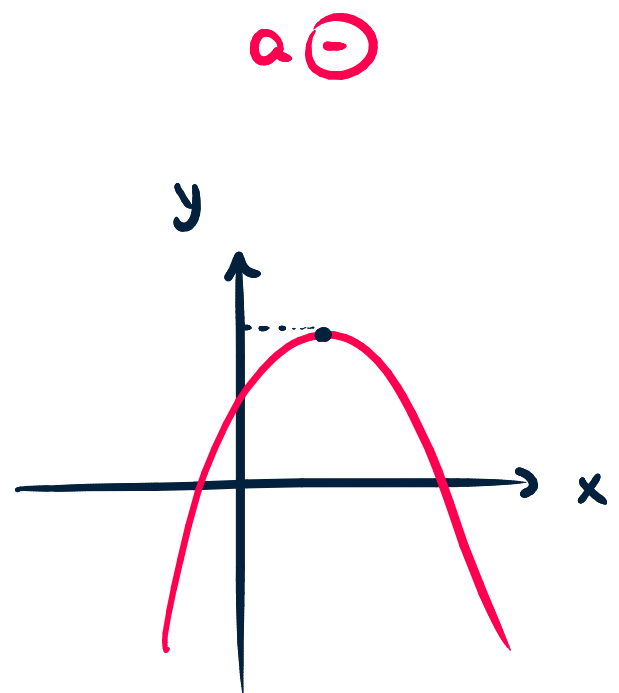
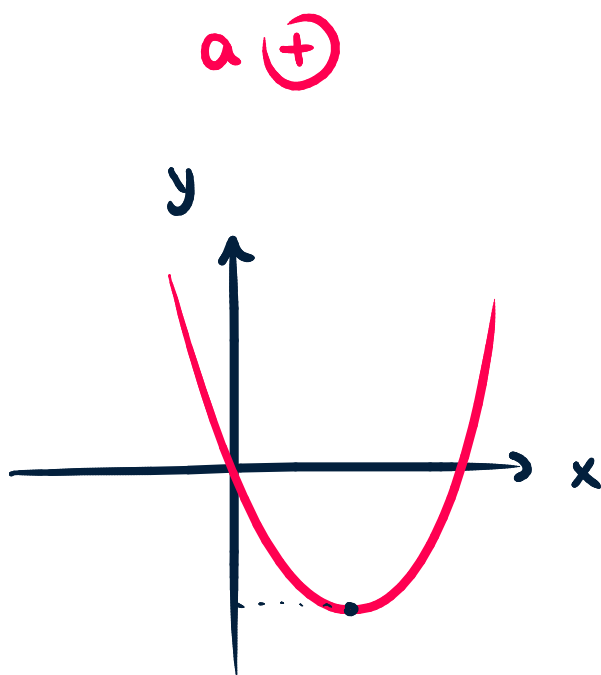
FORMA CANÔNICA

↳ CONCAVIDADE

$$f(x) = ax^2 + bx + c = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right]$$

Se $a > 0$: mínimo

Se $a < 0$: máximo



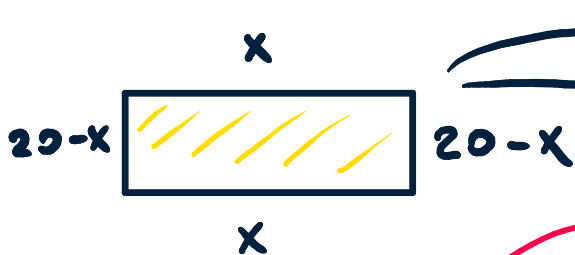
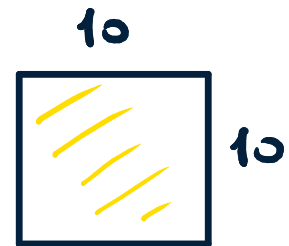
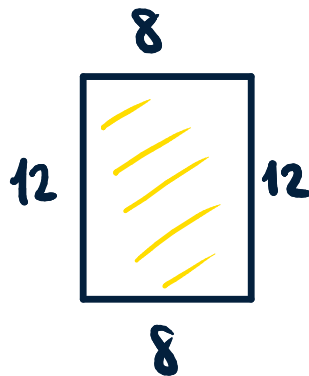
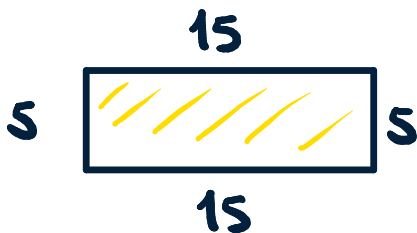
FORMA CANÔNICA

↳ MÁXIMO E MÍNIMO

EXEMPLO 01.

Dado um retângulo de perímetro 40, quais devem ser suas dimensões para que a área seja máxima?

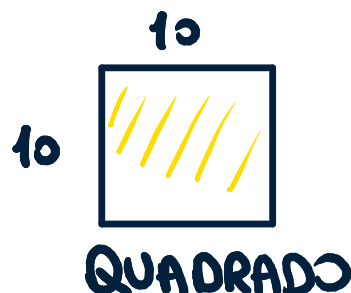
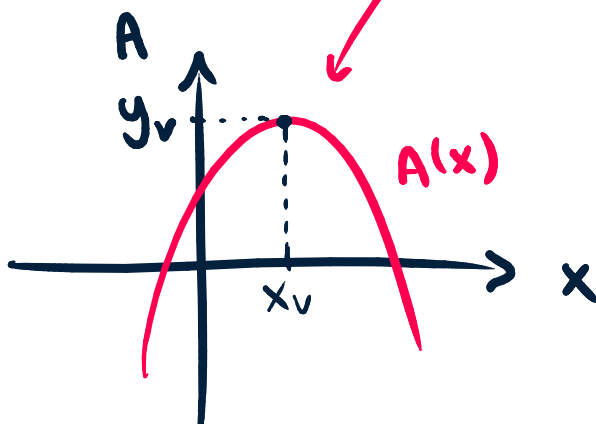
R:



$$A = b \cdot h = x(20 - x)$$

$$A(x) = -x^2 + 20x$$

$$i) x_v = -\frac{b}{2a} = \frac{-20}{-2} = 10 //$$



FORMA CANÔNICA

↳ MÁXIMO E MÍNIMO

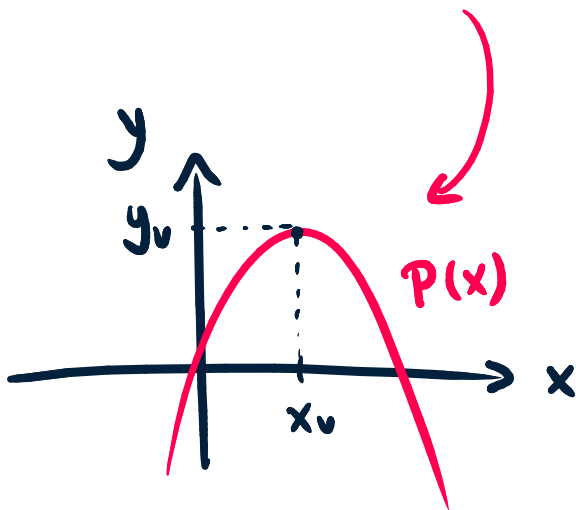
• EXEMPLO 02: SOMA E PRODUTO •

Dados dois números cuja soma é 10, qual é o maior valor para o seu produto?

R: $n_1 = x$
 $n_2 = 10 - x$

$$p = n_1 \cdot n_2 = x \cdot (10 - x) = 10x - x^2$$

$$p(x) = -x^2 + 10x$$



$$i) x_v = \frac{-b}{2a} = \frac{-10}{-2} = 5 //$$

$$\cdot n_1 = x = 5$$

$$\cdot n_2 = 10 - x = 5$$

$$ii) y_v = n_1 \cdot n_2 = 5 \cdot 5 = 25$$



FORMA CANÔNICA

↳ EIXO DE SÍMETRIA

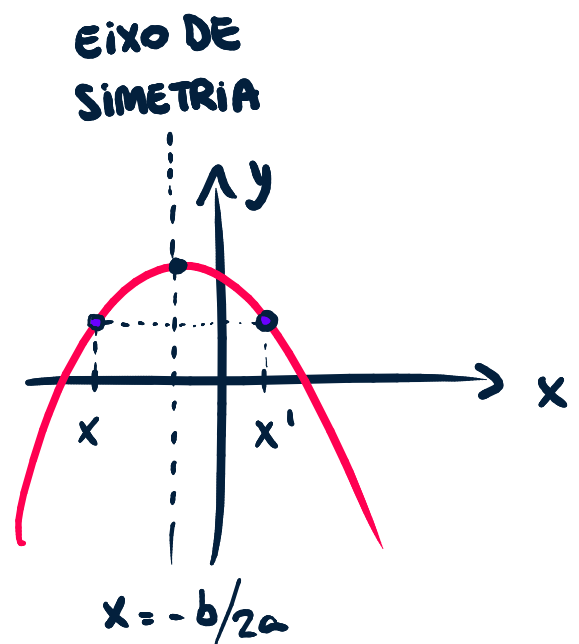
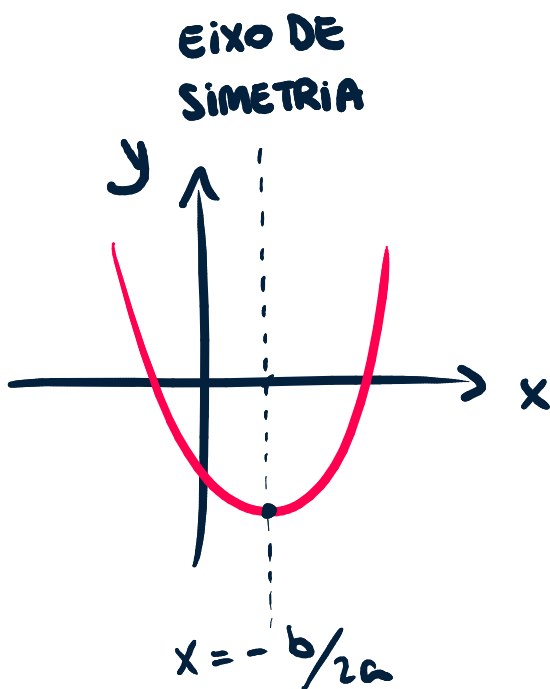
$$f(x) = ax^2 + bx + c = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right]$$

↳ Em qual ocasião temos $f(x) = f(x')$?

R: $f(x) = f(x')$

$$\left(x + \frac{b}{2a} \right)^2 = \left(x' + \frac{b}{2a} \right)^2$$

$$\left(x + \frac{b}{2a} \right) = - \left(x' + \frac{b}{2a} \right) \therefore \frac{x + x'}{2} = \frac{-b}{2a} = x_v$$



FORMA CANÔNICA

↳ EIXO DE SÍMETRIA

• EXEMPLO •

Esboce o gráfico de $f(x) = 2x^2 - 8x$

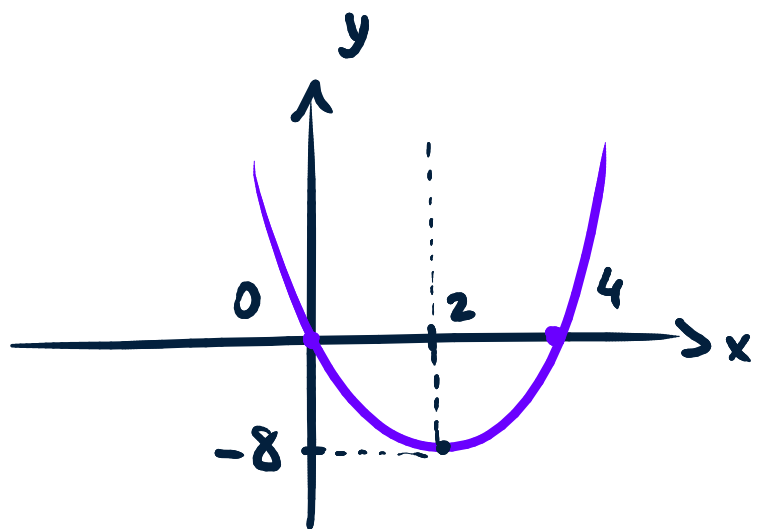
R :

$$f(x) = 2x^2 - 8x = 2x(x - 4)$$

i) RAÍZES : $f(x) = 0 \quad \therefore 2x(x - 4) = 0$

$\downarrow$ $\downarrow$
 $x=0$ $x=4$

ii) $x_v = \frac{-b}{2a} = \frac{8}{4} = 2$



SOMA e PRODUTO

Dados a soma e o produto de 2 números,
como encontra'-los ?



$$\underline{\text{SOMA}} : x_1 + x_2 = S \quad (\text{I})$$

$$\underline{\text{PRODUTO}} : x_1 \cdot x_2 = P \quad (\text{II})$$

$$(\text{I}) \rightarrow (\text{II}) : x_1 \cdot (S - x_1) = P$$

$$x_1 \cdot S - x_1^2 = P$$

$$x^2 - S \cdot x + P = 0$$

$$\underline{\text{EXEMPLO}} : x^2 - \underline{10}x + \underline{21} = 0$$

$$S = 10$$

$$P = 21$$

$$\text{RAÍZES} \begin{cases} x_1 = 3 \\ x_2 = 7 \end{cases}$$



SOMA e PRODUTO

EXEMPLO 01.

Encontre as raízes de $f(x) = x^2 - 5x + 6$

$$\underline{R} : \quad x^2 - 5x + 6 = 0$$

$$x^2 - S \cdot x + P = 0$$

$$\underline{RAÍZES} \quad \begin{cases} x_1 = 2 \\ x_2 = 3 \end{cases}$$



SOMA e PRODUTO

EXEMPLO 02.

Encontre as raízes de $f(x) = 2x^2 + 14x + 20$

$$\underline{R} : 2x^2 + 14x + 20 = 0$$

$$x^2 + 7x + 10 = 0$$

$\div 2$

$$x^2 - Sx + P = 0$$

$$x_1 + x_2 = -7$$

$$x_1 \cdot x_2 = 10$$

$$\underline{\text{RAÍZES}} : \begin{cases} x_1 = -2 \\ x_2 = -5 \end{cases}$$



FORMA FATORADA

$$f(x) = ax^2 + bx + c$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right]$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \left(\frac{\sqrt{\Delta}}{2a} \right)^2 \right]$$

$x^2 - y^2 = (x+y) \cdot (x-y)$

$$= a \left(x + \frac{b}{2a} + \frac{\sqrt{\Delta}}{2a} \right) \left(x + \frac{b}{2a} - \frac{\sqrt{\Delta}}{2a} \right)$$

$$= a \left[x - \left(\frac{-b - \sqrt{\Delta}}{2a} \right) \right] \left[x - \left(\frac{-b + \sqrt{\Delta}}{2a} \right) \right]$$

$$= a (x - r_1) (x - r_2),$$

onde r_1 e r_2 são as raízes.



FORMA FATORADA

EXEMPLO 01.

Escreva na forma fatorada a função

$$f(x) = 2x^2 - 12x + 10$$

$$\underline{R}: \quad \Delta = 144 - 4 \cdot 2 \cdot 10 = 144 - 80 = 64$$

$$x = \frac{12 \pm 8}{4} \quad \begin{array}{l} \nearrow r_1 = 5 \\ \searrow r_2 = 1 \end{array}$$

$$f(x) = 2x^2 - 12x + 10 = 2 \cdot (x - 1)(x - 5)$$



FORMA FATORADA

EXEMPLO 02.

Escreva na forma fatorada a função

$$f(x) = x^2 - 14x + 49$$

$$\underline{R}: f(x) = x^2 - 14x + 49$$

$$= (x - 7)^2$$

$$= (x - 7) \cdot (x - 7)$$

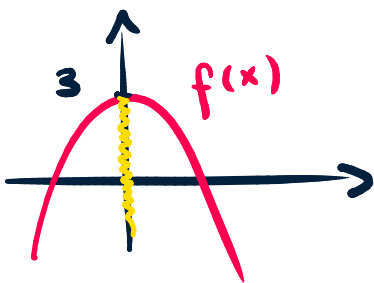


DOMÍNIO E IMAGEM

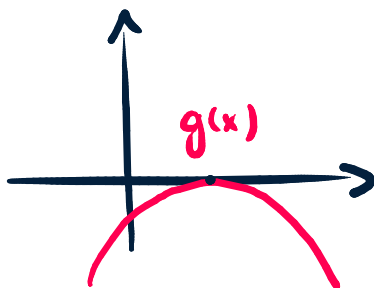
$$f(x) = ax^2 + bx + c$$

DOMÍNIO : $x \in \mathbb{R}$

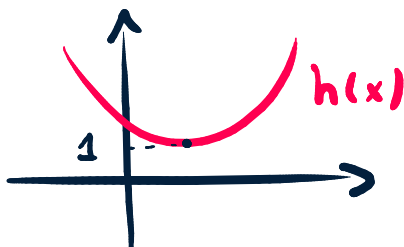
IMAGEM : é limitada pelo vértice



$$\text{Im}(f) = \{y \in \mathbb{R} \mid y \leq 3\}$$



$$\text{Im}(g) = \{y \in \mathbb{R} \mid y \leq 0\}$$



$$\text{Im}(h) = \{y \in \mathbb{R} \mid y \geq 1\}$$



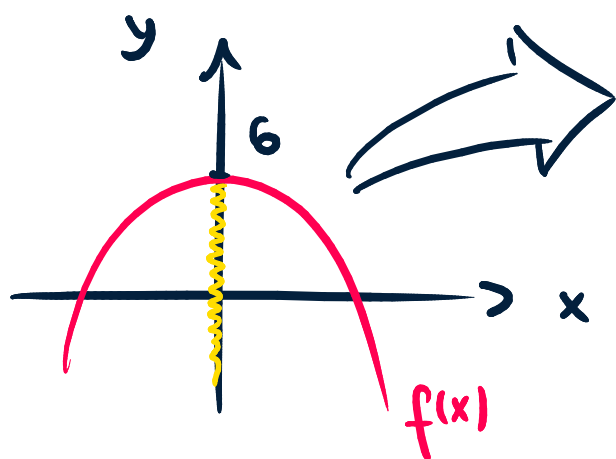
DOMÍNIO E IMAGEM

EXEMPLO 01.

Determine a imagem da função $f(x) = -x^2 + 2x + 5$

$$\underline{R}: \quad x_v = \frac{-b}{2a} = \frac{-2}{-2} = 1$$

$$y_v = f(1) = -1^2 + 2 \cdot 1 + 5 = 6$$



$$\text{Im}(f) = \{y \in \mathbb{R} \mid y \leq 6\}$$



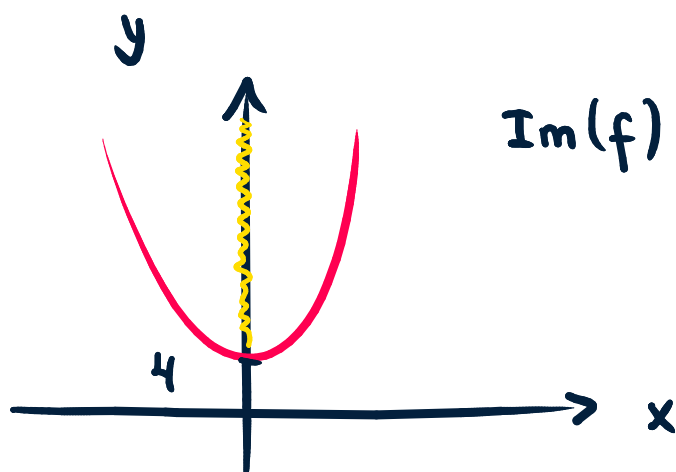
DOMÍNIO E IMAGEM

EXEMPLO 02.

Determine a imagem da função $f(x) = x^2 + 4$

$$\underline{R}: \quad x_v = -\frac{b}{2a} = \frac{-0}{2a} = 0$$

$$y_v = f(x_v) = f(0) = 4$$



$$\text{Im}(f) = \{y \in \mathbb{R} \mid y \geq 4\}$$

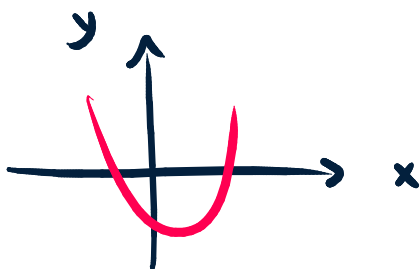


CONSTRUINDO GRÁFICOS

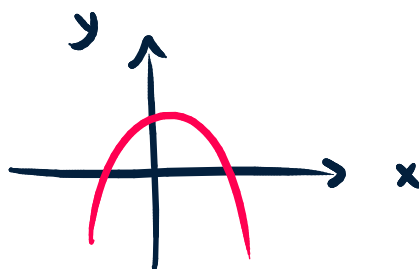
$$f(x) = ax^2 + bx + c$$

COEFICIENTE "a" : determina a concavidade

$a \oplus$

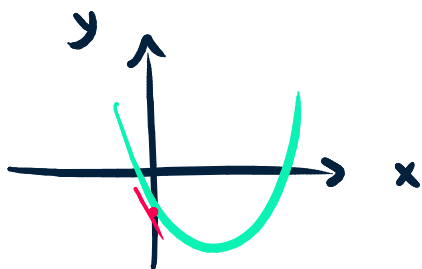


$a \ominus$

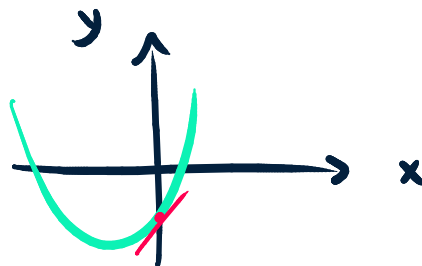


COEFICIENTE "b" :

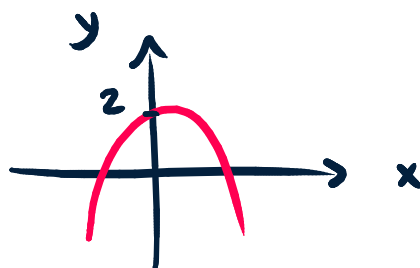
$b \ominus$



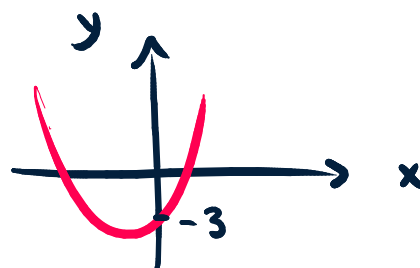
$b \oplus$



COEFICIENTE "c" : determina a interseção de eixo y



$a \ominus$
 $c = 2$



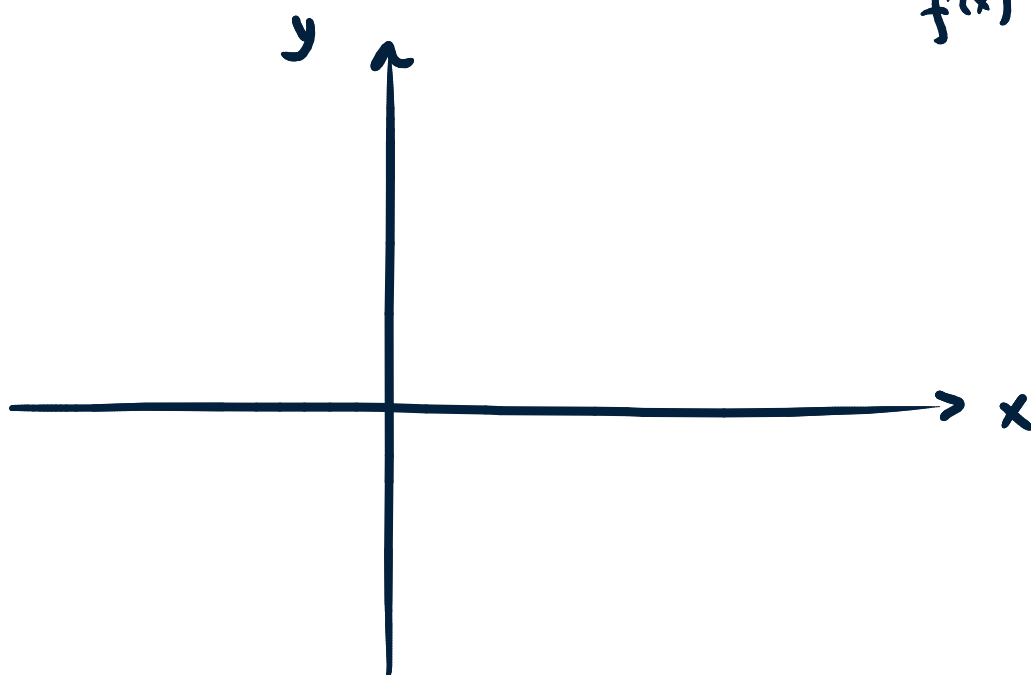
$a \oplus$
 $c = -3$



UNIVERSO NARRADO

PROTOCOLO:

- i) DETERMINAR CONCAVIDADE
- ii) ENCONTRAR O VÉRTICE
- iii) ENCONTRAR RAÍZES



$$f(x) = a x^2 + b x + c$$



CONSTRUINDO GRÁFICOS

EXEMPLO 01

Construa o gráfico da função $f(x) = x^2 - 9$

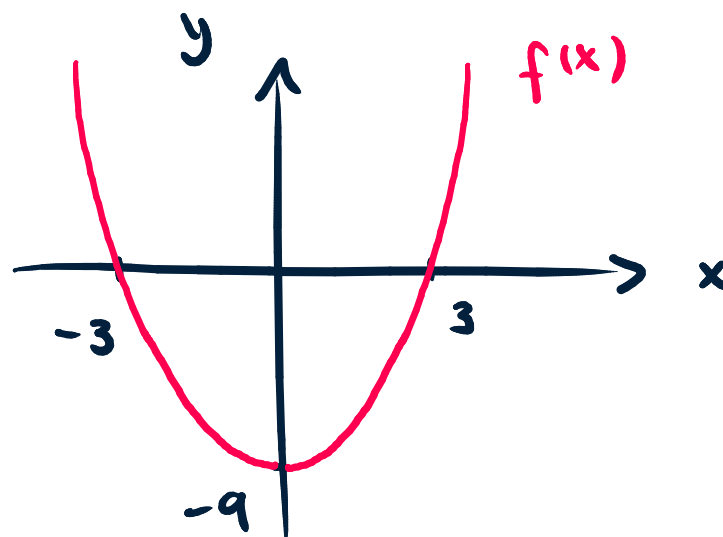
R :

i) $a \oplus$

ii) $\left. \begin{array}{l} x_1 = 3 \\ x_2 = -3 \end{array} \right\} \text{RAÍZES}$

iii) $x_v = \frac{-b}{2a} = 0$

$$y_v = f(x_v) = f(0) = -9$$



CONSTRUINDO GRÁFICOS

EXEMPLO 02.

Construa o gráfico da função $f(x) = -2x^2 + 10x - 8$

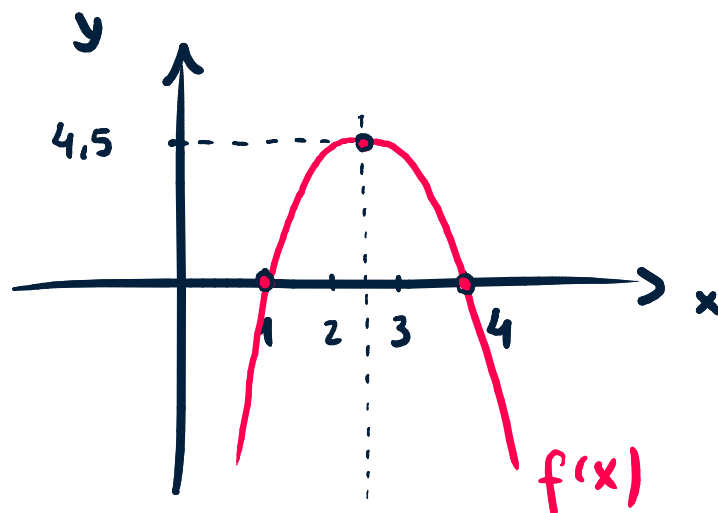
R :

i) $a \ominus$: $\cap$

ii) $\Delta = 100 - 4(-2)(-8) = 36$ $\begin{cases} x_1 = 4 \\ x_2 = 1 \end{cases}$

iii) $x_v = \frac{-b}{2a} = \frac{-10}{-4} = 2,5$

$$y_v = \frac{-\Delta}{4a} = \frac{-36}{4(-2)} = 4,5$$



$x = 2,5$

INEQUAÇÕES DO 2º GRAU

EXEMPLO 01.

Resolva a inequação $x^2 - 4x + 3 \leq 0$

R:

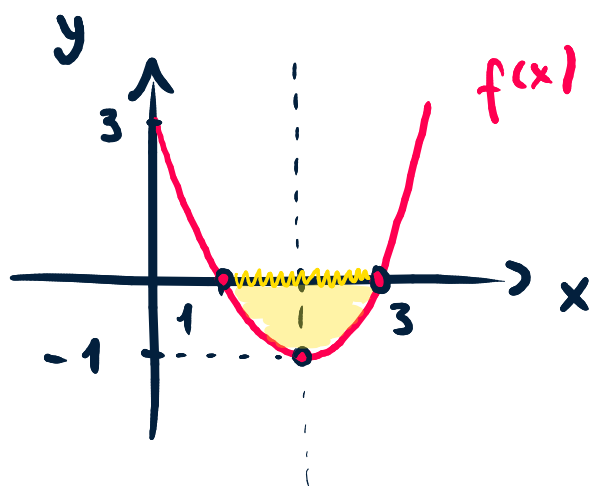
$$f(x) = x^2 - 4x + 3$$

i) a $\oplus$: $\cup$

ii) $x_1 = 1$ e $x_2 = 3$ (raízes)

iii) $x_v = \frac{4}{2} = 2_{//}$

$$y_v = f(x_v) = f(2) = -1_{//}$$



Solução :

$$x \in \mathbb{R} \mid 1 \leq x \leq 3$$



INEQUAÇÕES DO 2º GRAU

EXEMPLO 02 .

Resolva a inequação $-x^2 + 6x + 9 > 0$

R : $f(x) = -x^2 + 6x + 9$

i) $a \ominus : \wedge$

ii) $\Delta = 36 - 4 \cdot (-1) \cdot 9 = 36 \cdot 2$

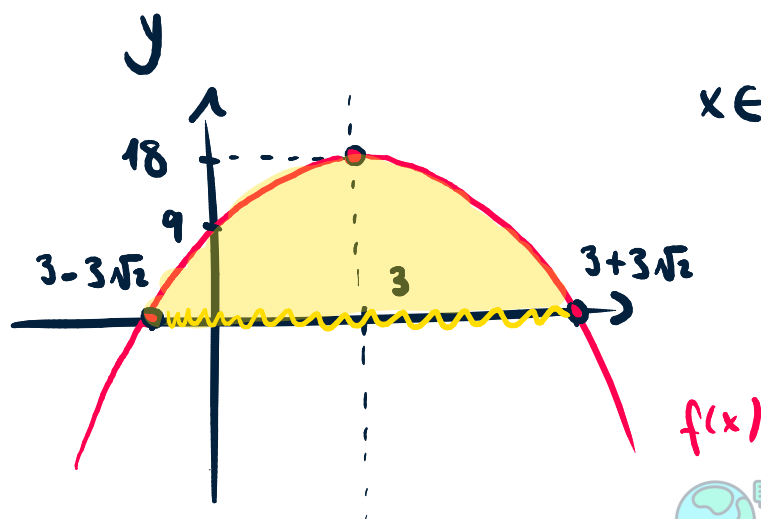
$\nearrow x_1 = 3 + 3\sqrt{2}$
 $\searrow x_2 = 3 - 3\sqrt{2}$

iii) $x_v = \frac{x_1 + x_2}{2} = \frac{6}{2} = 3_{//}$

$y_v = f(3) = 18$

Solução :

$x \in \mathbb{R} \mid 3 - 3\sqrt{2} < x < 3 + 3\sqrt{2}$



INEQUAÇÕES DO 2º GRAU

EXEMPLO 03.

Resolva a inequação $\frac{x^2 - 4}{x - 1} \leq 0$

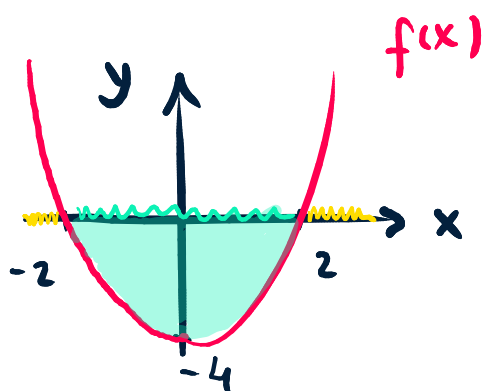
R: $f(x) = x^2 - 4$

i) a $\oplus$: $\cup$

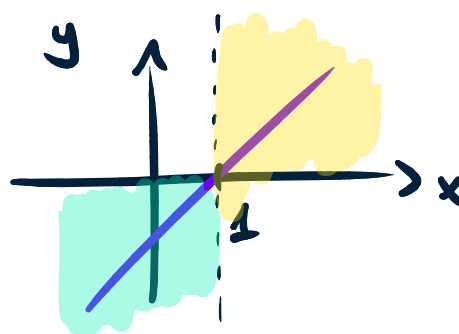
ii) Raízes: $\begin{cases} x_1 = 2 \\ x_2 = -2 \end{cases}$

iii) $x_v = 0$

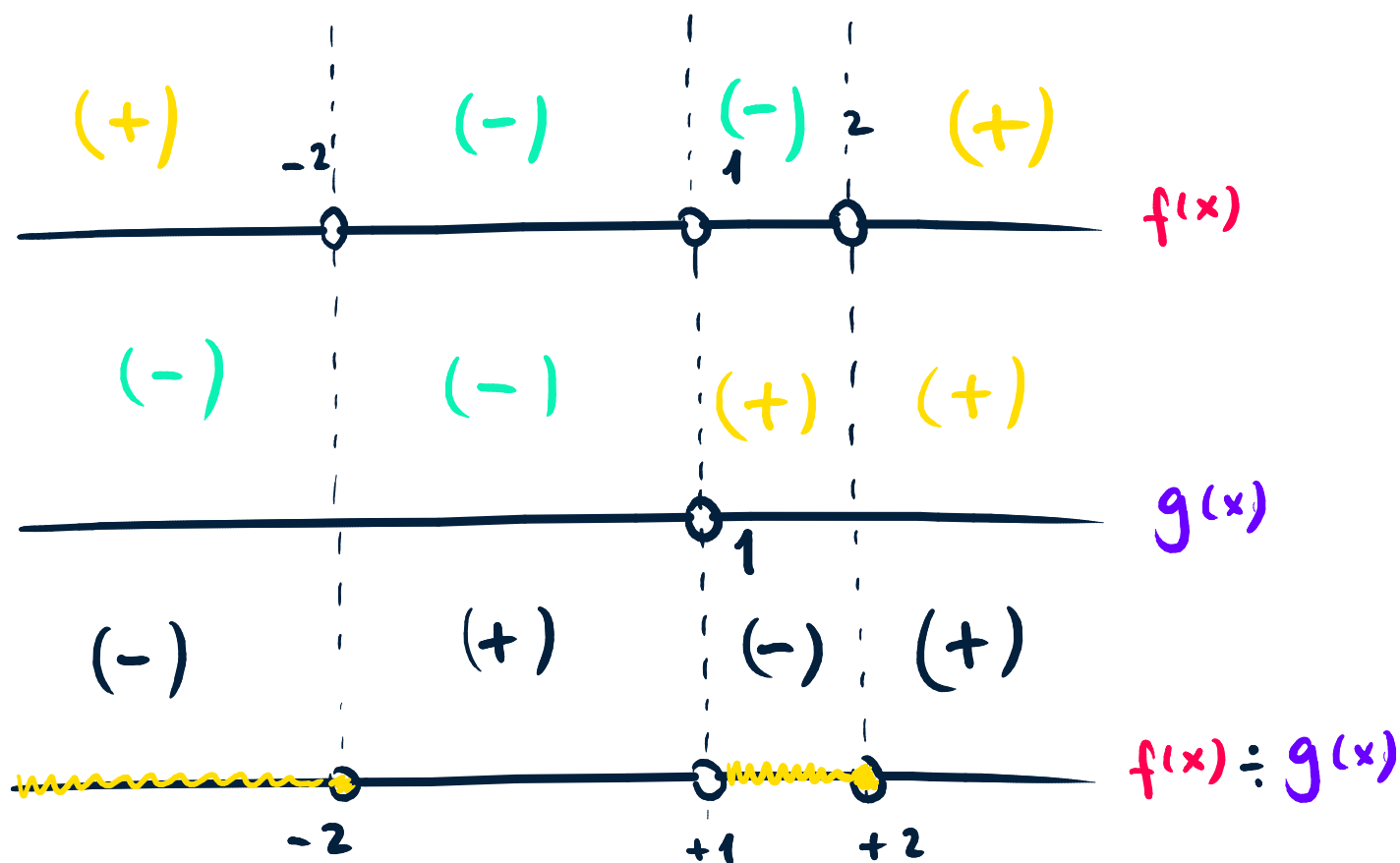
$y_v = f(0) = -4$



$g(x) = x - 1$



QUADRO DE SINAIS



$$\text{Solução} = \{ x \in \mathbb{R} \mid x \leq -2 \text{ ou } 1 < x \leq 2 \}$$

