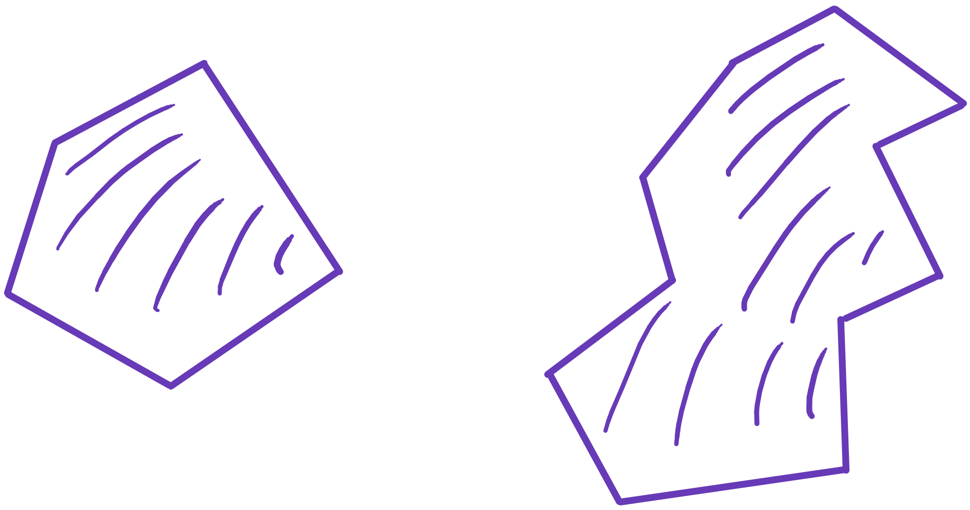
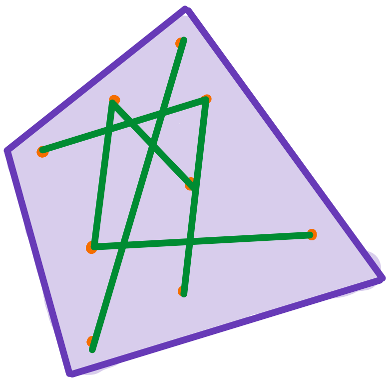
POLÍGONOS

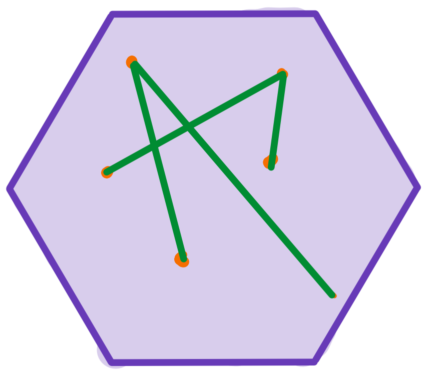
DEFINIÇÃO

POLÍGONOS SÃO FIGURAS PLANAS FECHADAS CUJOS LADOS SÃO SEGMENTOS DE RETA.

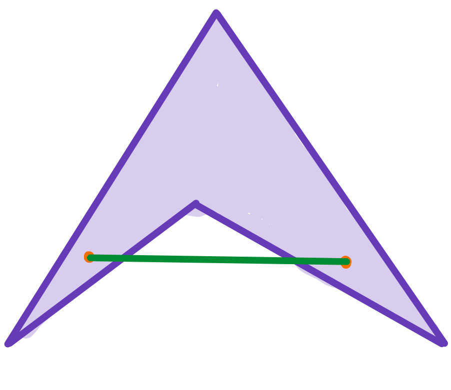


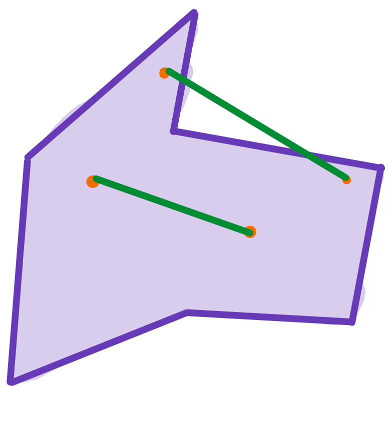
POLÍGONO CONVEXO





POLÍGONO CÔNCAVO





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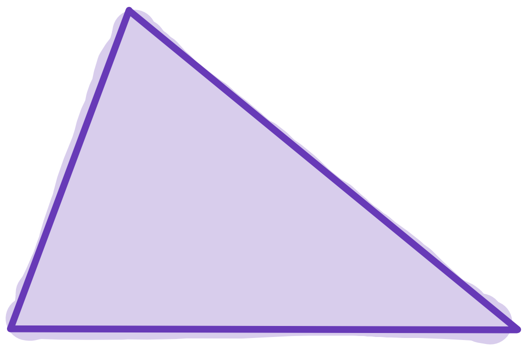
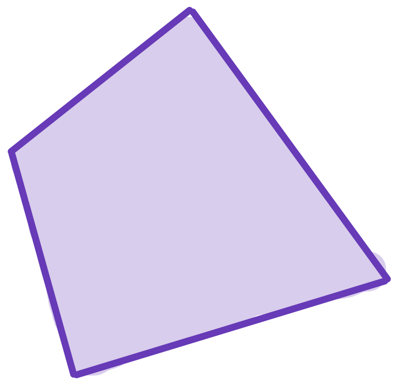
CLASSIFICAÇÃO

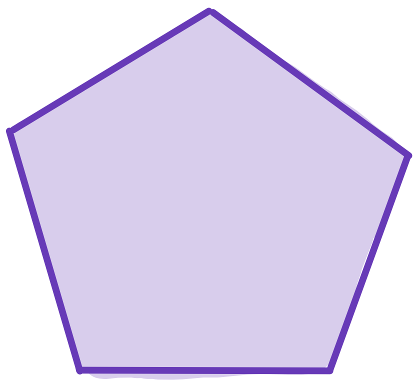
TRIÂNGULO



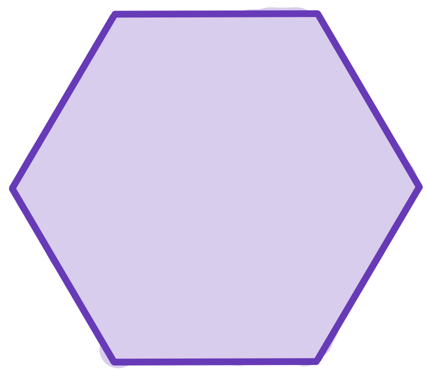
QUADRILÁTERO



PENTÁGONO

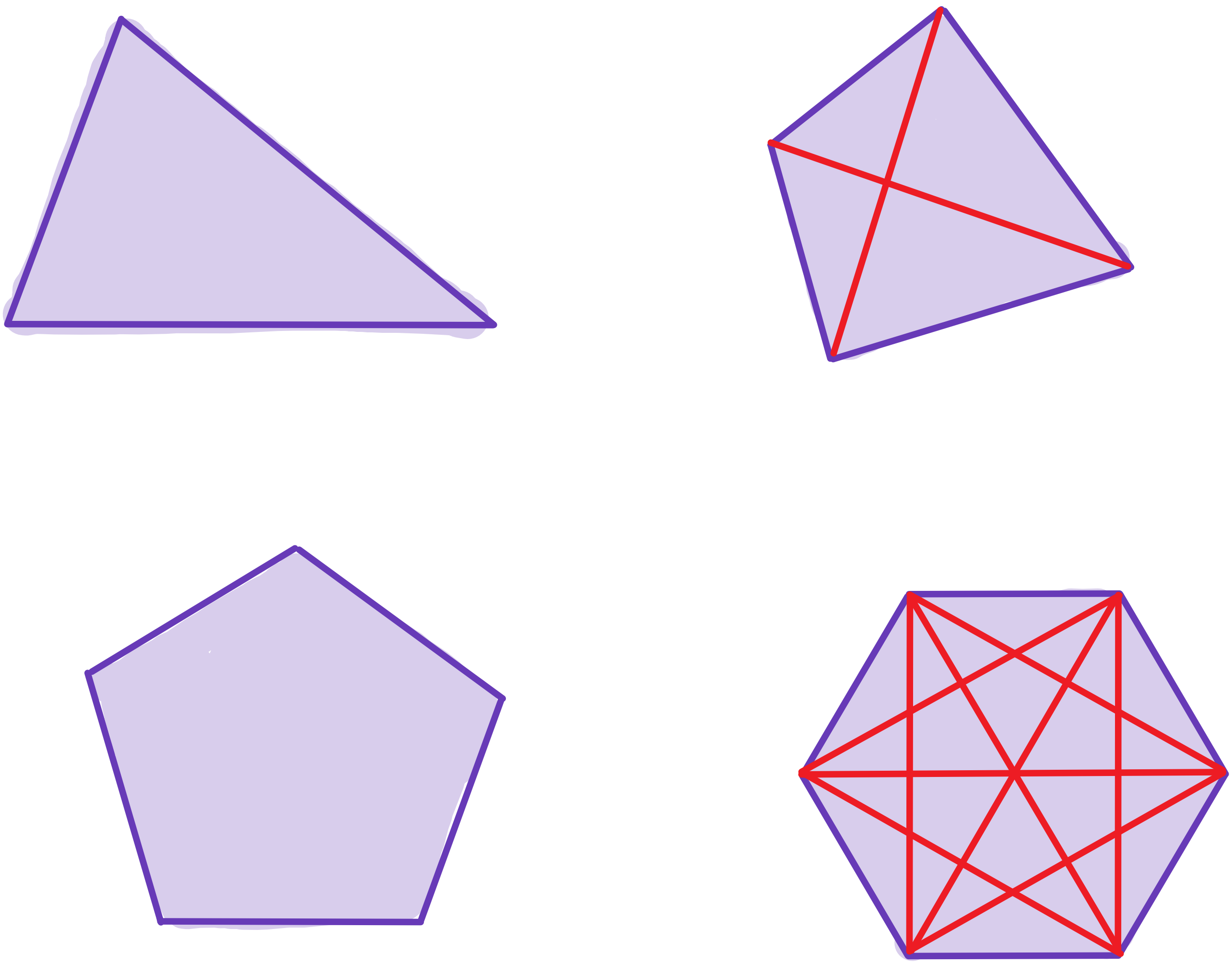


HEXÁGONO

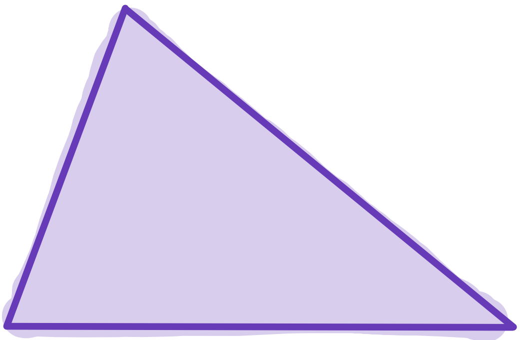


Nº LADOS	NOME
3	TRIÂNGULO
4	QUADRILÁTERO
5	PENTÁGONO
6	HEXÁGONO
7	HEPTÁGONO
8	OCTÓGONO
9	ENEÁGONO
10	DECÁGONO
11	UNDECÁGONO
12	DODECÁGONO
20	ICOSÁGONO

NÚMERO DE DIAGONAIS

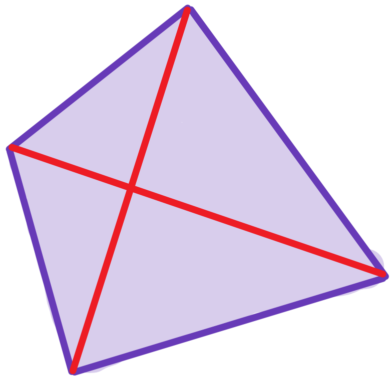


Polígono n lados : $\boxed{d = \dfrac{n(n-3)}{2}}$



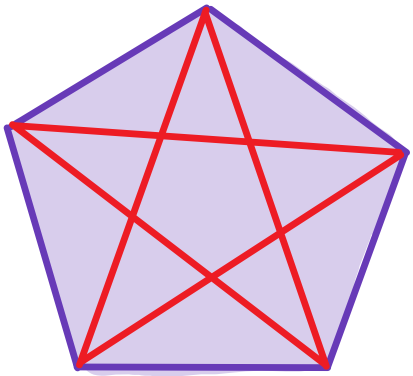
$$d = \frac{3(3-3)}{2}$$

$$d = 0$$



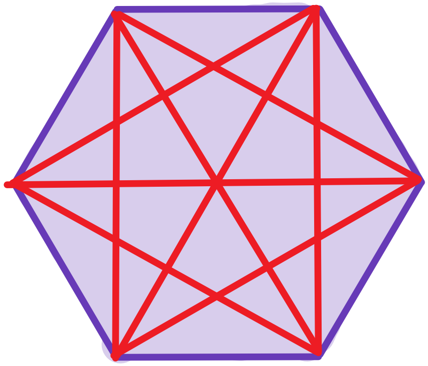
$$d = \frac{4(4-3)}{2}$$

$$d = 2$$



$$d = \frac{5(5-3)}{2}$$

$$d = 5$$



$$d = \frac{\overset{3}{\cancel{6}}(6-3)}{\cancel{2}}$$

$$d = 9$$

EXEMPLO

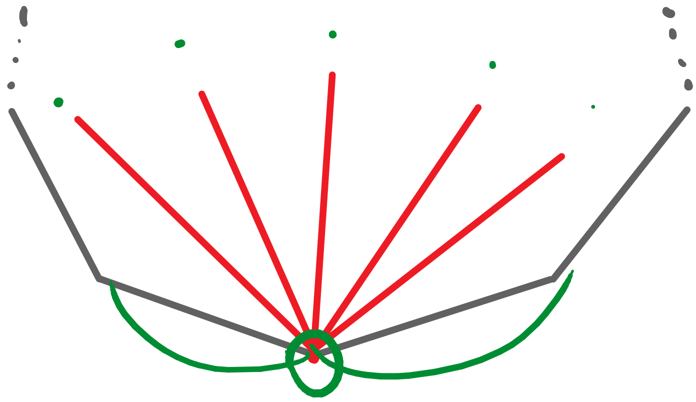
EM UM POLÍGONO REGULAR, O NÚMERO DE DIAGONAIS QUE PARTEM DE CADA VÉRTICE É IGUAL AO NÚMERO TOTAL DE DIAGONAIS DE UM PENTÁGONO.

CALCULE O NÚMERO DE LADOS DESSE POLÍGONO.

DIAG. PENTÁGONO :

$$d = \frac{n(n-3)}{2}$$

$$d = \frac{5(5-3)}{2} = 5$$



$n = 5 + 3 = 8$

OCTÓGONO

EXEMPLO

AO AUMENTAR O NÚMERO DE LADOS DE UM POLÍGONO EM 2 UNIDADES, O NÚMERO DE DIAGONAIS AUMENTA 15 UNIDADES.

CALCULE O NÚMERO DE DIAGONAIS DO POLÍGONO INICIAL.

I $\qquad\qquad\qquad$ II

$n \rightarrow 8 \qquad\qquad\qquad n+2 \rightarrow 10$

$$d_I = \frac{n(n-3)}{2} \quad ; \quad d_{II} = \frac{(n+2)(n+2-3)}{2}$$

$$\xrightarrow{+15} \quad d_{II} = \frac{(n+2)(n-1)}{2}$$

$$d_{II} - d_I = 15$$

$$\frac{(n+2)(n-1)}{\cancel{2}} - \frac{n(n-3)}{\cancel{2}} = \frac{30}{\cancel{2}}$$

$$\cancel{n^2} - n + 2n - 2 - \cancel{n^2} + 3n = 30$$

$$4n = 32$$

$$n = 8$$

$$d = \frac{\overset{4}{\cancel{8}}(8-3)}{\cancel{2}} \longrightarrow d_I = 20$$

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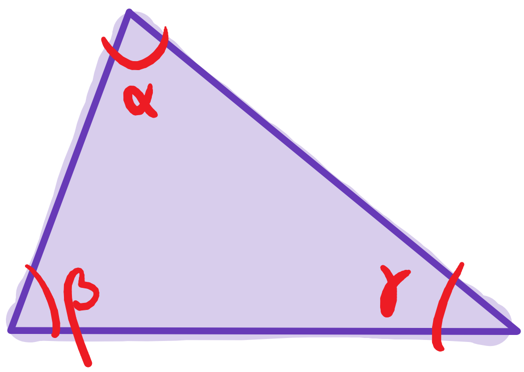
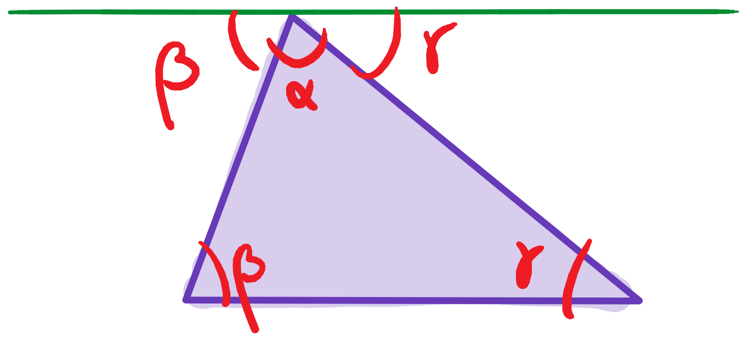
SOMA DOS ÂNGULOS INTERNOS

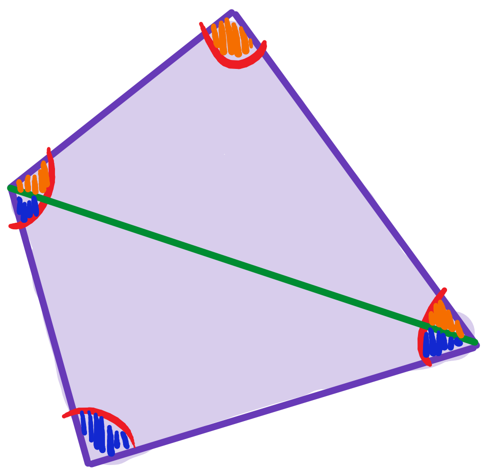
TRIÂNGULO





$S_i = 180°$

QUADRILÁTERO

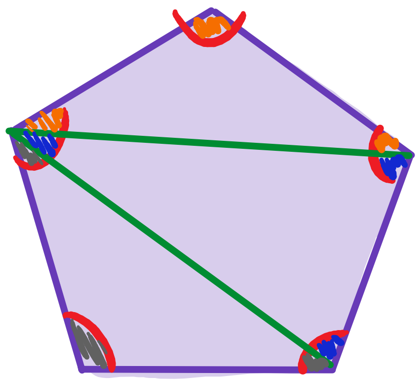


2 TRIÂNGULOS

$S_i = 180° \cdot 2$

$S_i = 360°$

PENTÁGONO

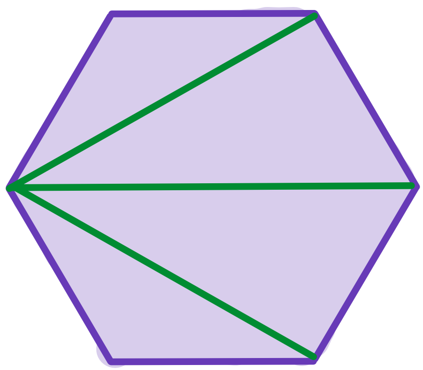


3 TRIÂNGULOS

$S_i = 180° . 3$

$S_i = 540°$

HEXÁGONO



4 TRIÂNGULOS

$S_i = 180° . 4$

$S_i = 720°$

POLÍGONO DE n LADOS

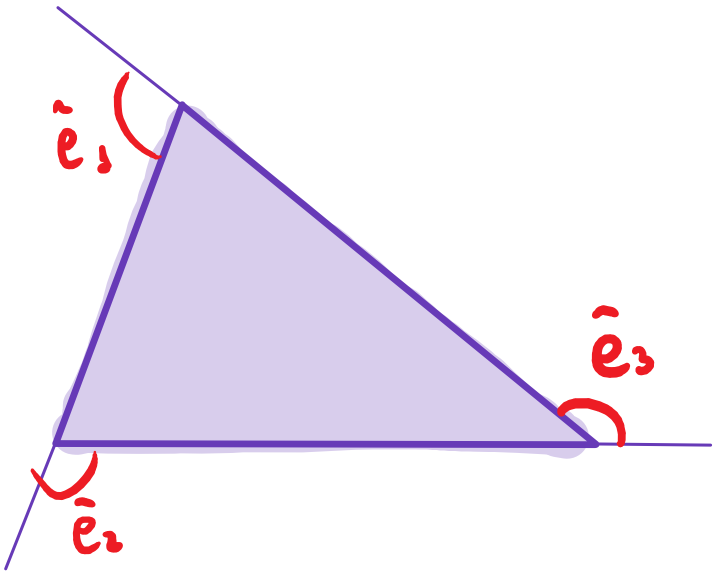
FORMADO POR : $n - 2$ TRIÂNGULOS

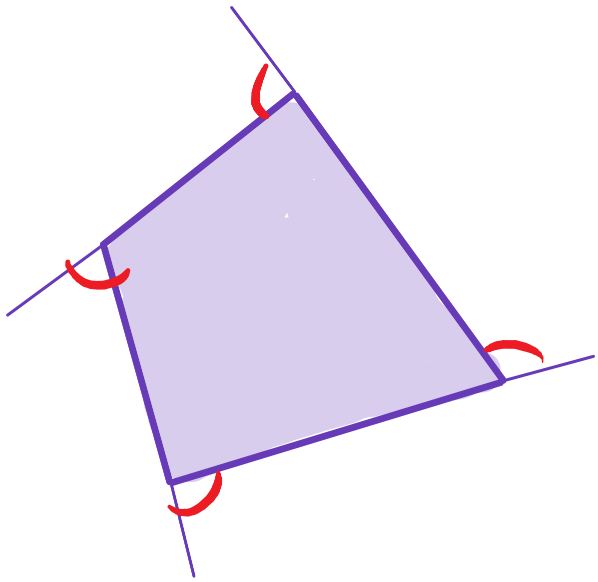
$$\boxed{S_i = 180^\circ(n-2)}$$

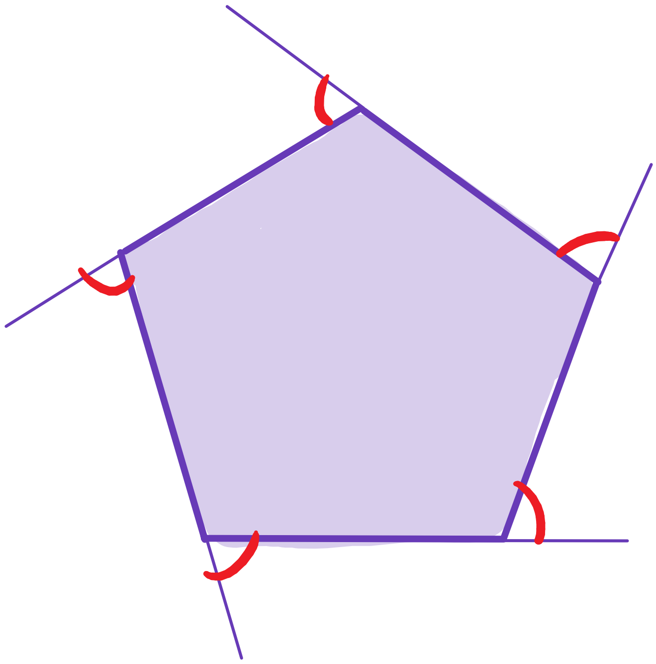
$n = 6 \rightarrow S_i = 180 . (6-2) = 720^\circ$

$n = 12 \rightarrow S_i = 180 (12-2) = 1800^\circ$

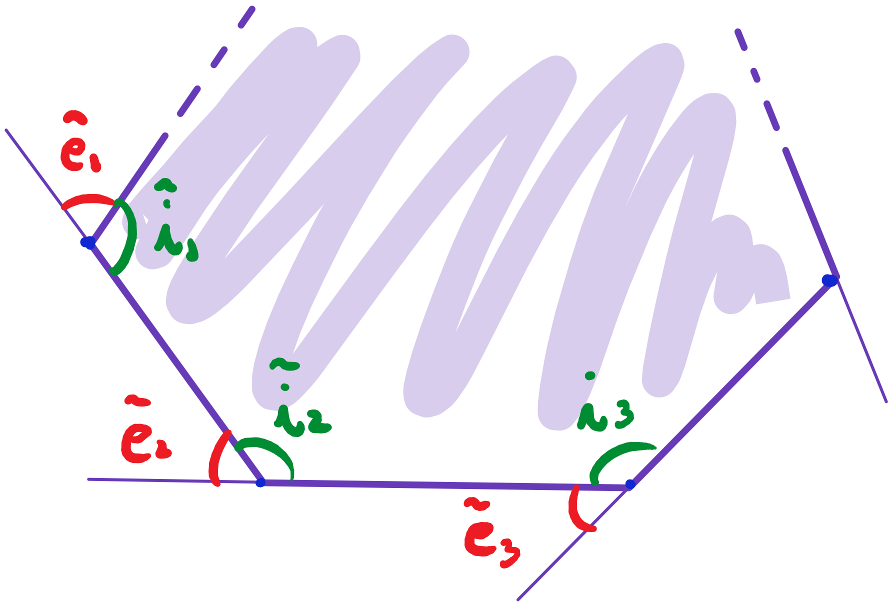
SOMA DOS ÂNGULOS EXTERNOS







POLÍGONO DE n LADOS



$$S_i + S_e = 180 \cdot n$$

$$180(n-2) + S_e = 180\,n$$

$$\cancel{180n} - 360^\circ + S_e = \cancel{180n}$$

$$\boxed{S_e = 360^\circ}$$

EXEMPLO

QUAL A SOMA DOS ÂNGULOS INTERNOS DE UM POLÍGONO QUE POSSUI 35 DIAGONAIS?

$$d = \frac{n(n-3)}{2}$$

$$35 = \frac{n(n-3)}{2}$$

$$n(n-3) = 70$$

$$10 \cdot 7 = 70$$

$$\underline{n = 10}$$

$$S_i = 180(n-2)$$

$$S_i = 180(10-2)$$

$$\underline{S_i = 1.440^\circ}$$

EXEMPLO

AS MEDIDAS DOS ÂNGULOS INTERNOS DE UM PENTÁGONO SÃO DADAS POR: x, $x+15$, $2x-10$, $2x-5$ E $2x+20$.

QUAL A MEDIDA DO MAIOR ÂNGULO EXTERNO?

$$S_i = 180(5-2) \rightarrow S_i = 540°$$

$$x + x + \cancel{15} + 2x - \cancel{10} + 2x \cancel{-5} + 2x + 20 = 540°$$

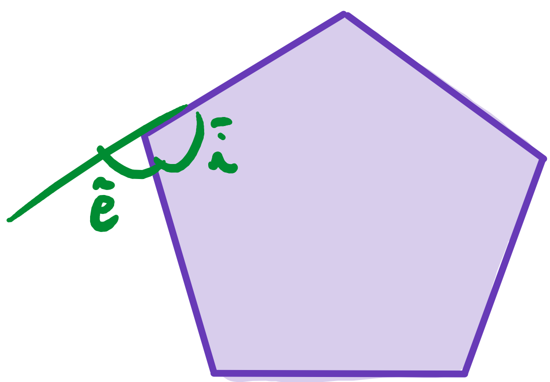
$$8x = 520$$

$$x = 65°$$

$$\hat{e}_1 = 180 - 65 = 115°$$

$\hat{i}_1 = 65°$; $\hat{i}_2 = 80°$ $\hat{i}_3 = 120°$

$\hat{i}_4 = 125°$ $i_5 = 150°$



EXEMPLO

DOIS ÂNGULOS INTERNOS DE UM POLÍGONO CONVEXO MEDEM 130° CADA E OS DEMAIS MEDEM 128° CADA.

CALCULE O NÚMERO DE LADOS DESSE POLÍGONO.

$$n \begin{cases} \rightarrow 2 \cdot 130^\circ \\ \rightarrow (n-2)\,128^\circ \end{cases} \Bigg\} \; S_i = 180(n-2)$$

$$128(n-2) + 130 \cdot 2 = 180(n-2)$$

$$260 = 180(n-2) - 128(n-2)$$

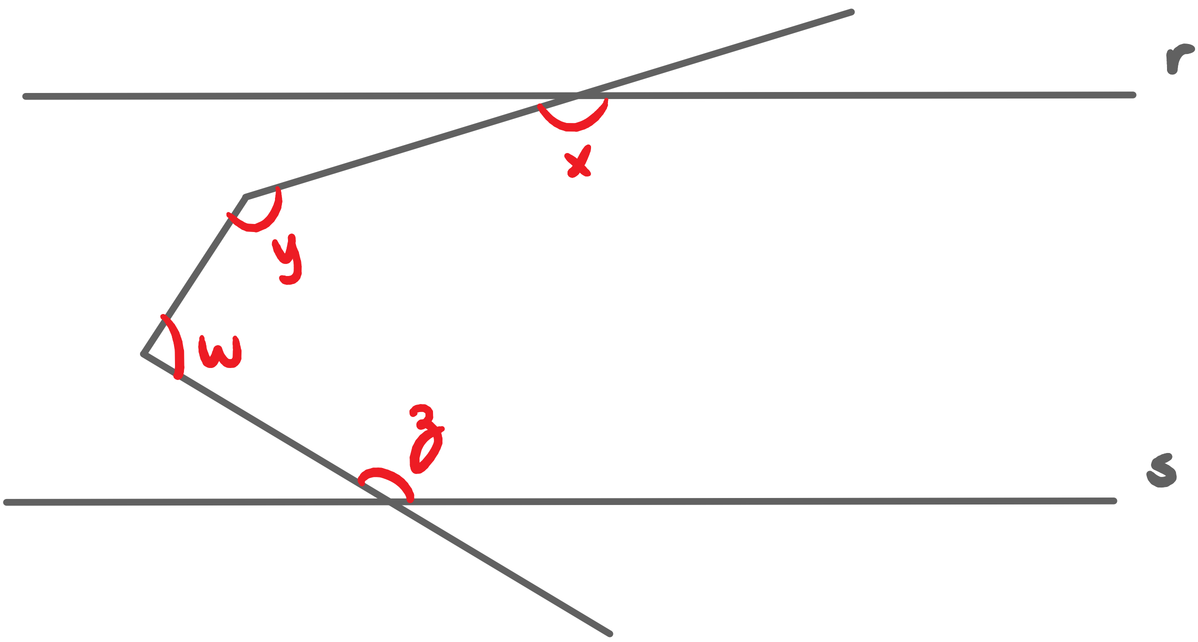
$$\overset{5}{\cancel{260}} = \cancel{52}(n-2)$$

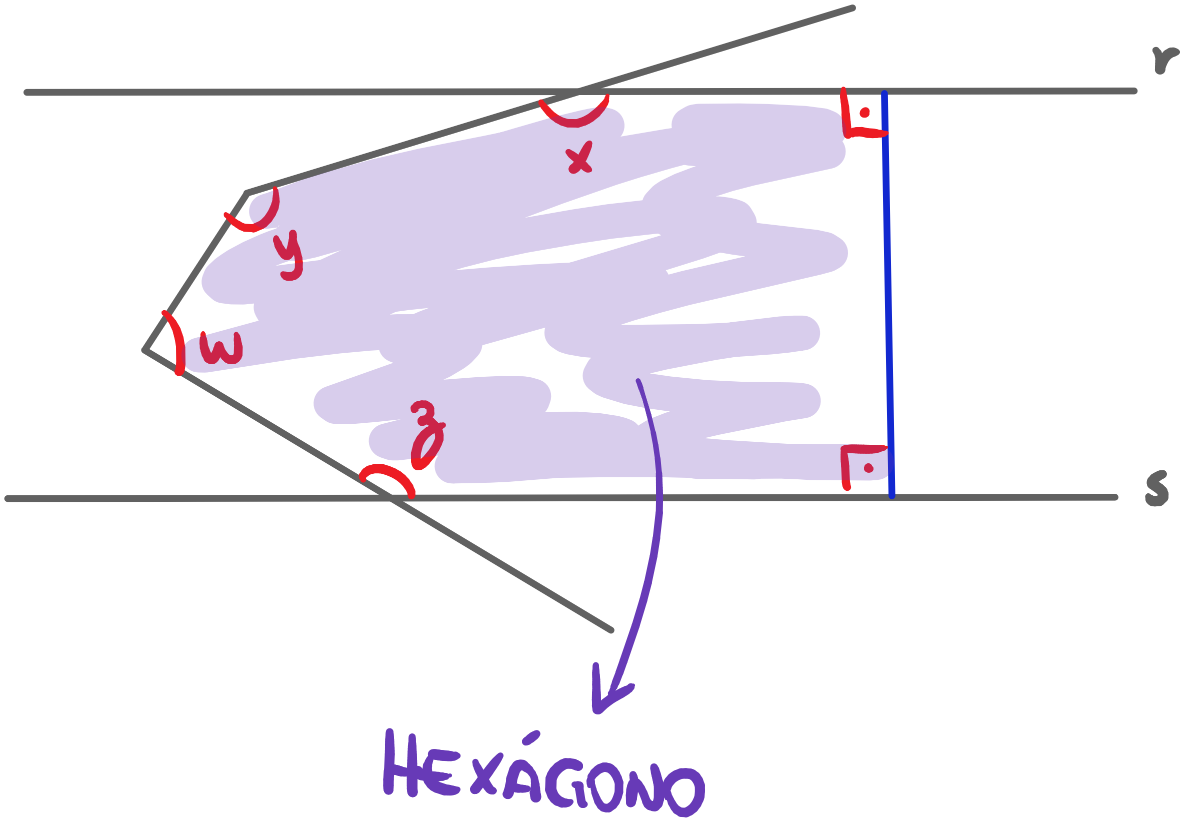
$$n-2 = 5$$

$$\underline{n = 7}$$

EXEMPLO

SABENDO QUE AS RETAS r E s SÃO PARALELAS, DETERMINE O VALOR DA SOMA DOS ÂNGULOS INDICADOS POR x, y, w E z.





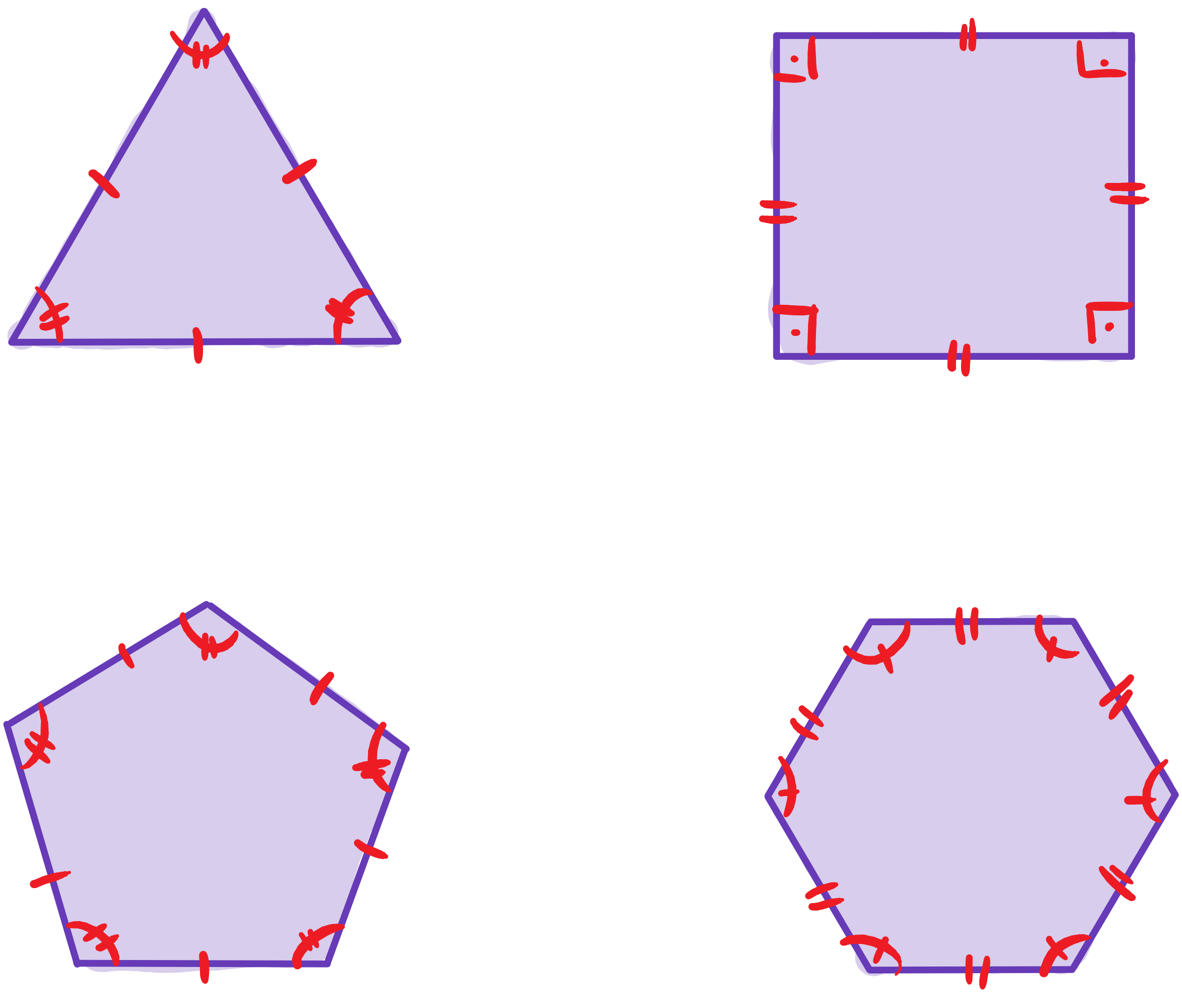
$$x+y+w+z+\underbrace{90^\circ+90^\circ}_{180^\circ}=\underbrace{180(6-2)}_{720^\circ}$$

$$x+y+w+z=720^\circ-180^\circ$$

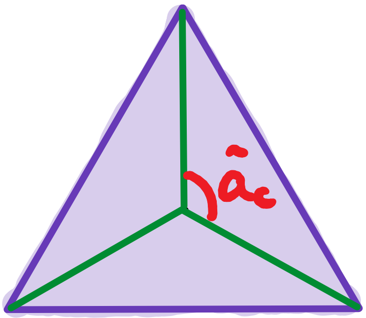
$$\underline{x+y+w+z=540^\circ}$$

POLÍGONO REGULAR

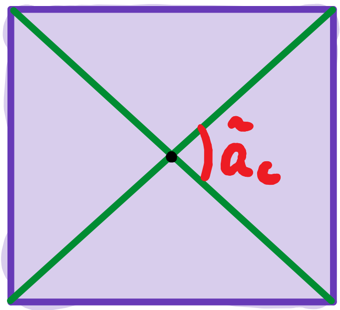
POLÍGONOS REGULARRES SÃO POLÍGONOS QUE POSSUEM TODOS OS LADOS CONGRUENTES (EQUILÁTERO), ASSIM COMO TODOS OS ÂNGULOS (EQUIÂNGULO).



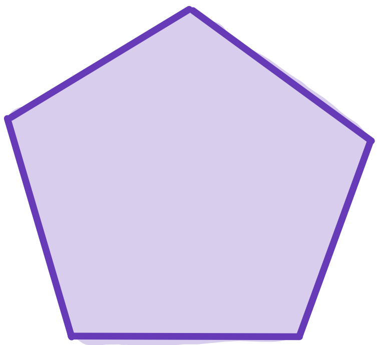
ÂNGULO CENTRAL



$$\hat{a}_c = \frac{360°}{3} = 120°$$



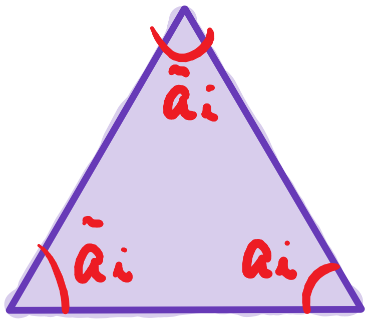
$$a_c = \frac{360°}{4} = 90°$$



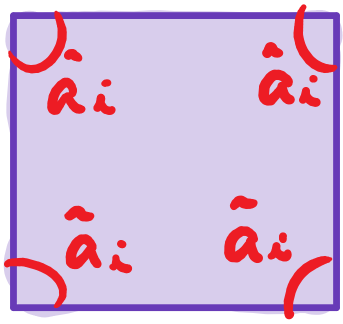
$$a_c = \frac{360°}{5} = 72°$$

n LADOS : $\boxed{\bar{a}_c = \frac{360°}{n}}$

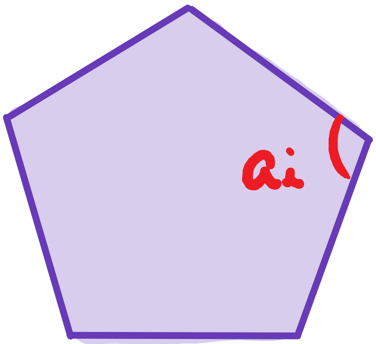
ÂNGULO INTERNO



$$\hat{a}_i = \frac{180°}{3} = 60°$$



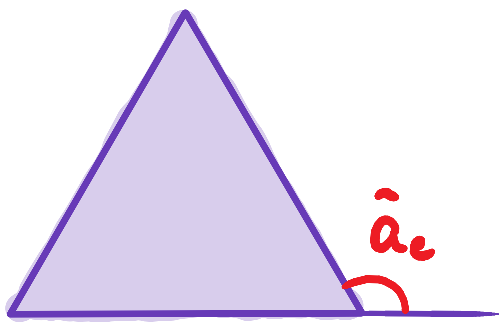
$$a_i = \frac{360°}{4} = 90°$$



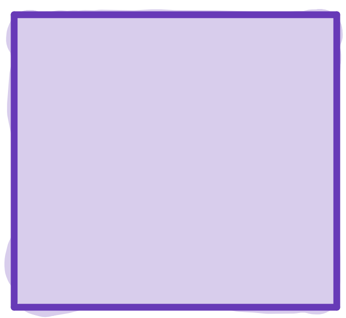
$$a_i = \frac{540°}{5} = 108°$$

n LADOS : $\boxed{\hat{a}_i = \frac{S_i}{n} = \frac{180(n-2)}{n}}$

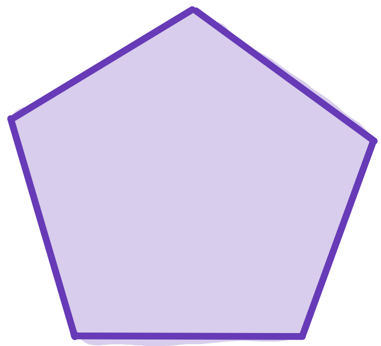
ÂNGULO EXTERNO



$$\hat{a}_e = \frac{360°}{3} = 120°$$



$$\hat{a}_e = \frac{360°}{4} = 90°$$



$$\hat{a}_e = \frac{360°}{5} = 72°$$

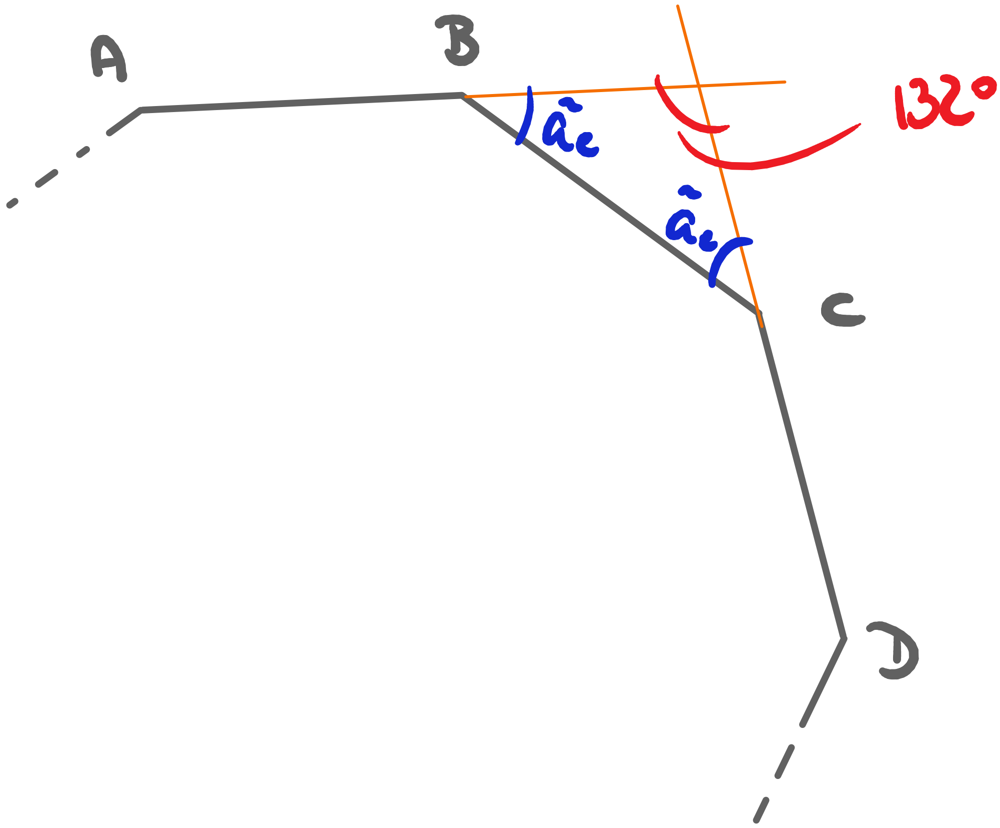
n LADOS :

$$\boxed{\hat{a}_e = \frac{S_e}{n} = \frac{360°}{n}}$$

EXEMPLO

SEJA O POLÍGONO REGULAR ABCDE... O PROLONGAMENTO DOS LADOS AB E CD FORMAM UM ÂNGULO DE 132°.

DETERMINE QUAL O POLÍGONO EM QUESTÃO.



$$2\hat{a}_e + 132° = 180°$$

$$\hat{a}_e = \frac{48°}{2}$$

$$\hat{a}_e = 24°$$

$$\hat{a}_e = \frac{S_e}{n} \rightarrow 24° = \frac{360°}{n} \rightarrow n = \frac{360}{24}$$

$$n = 15$$

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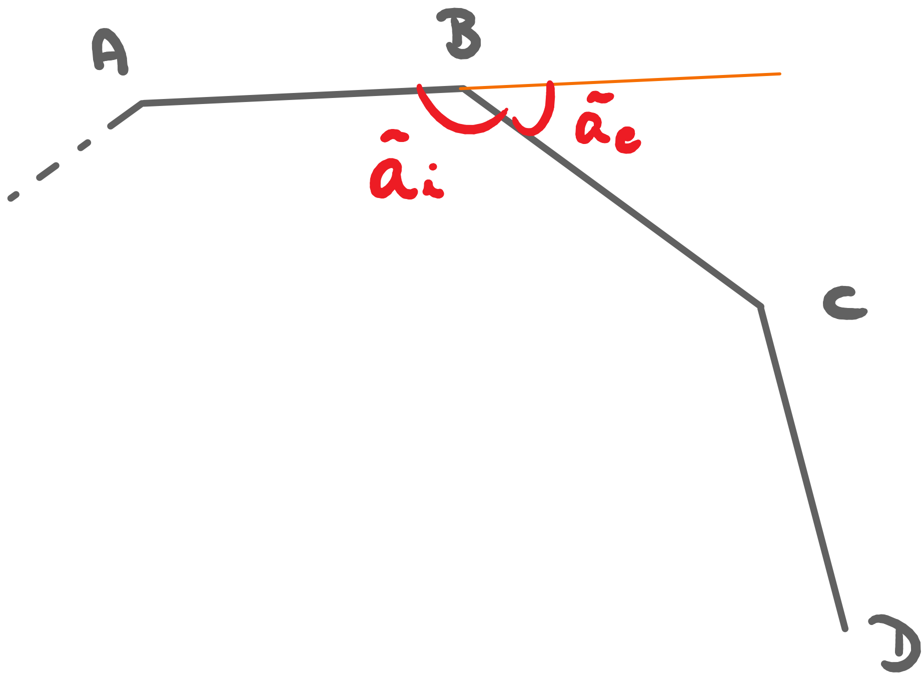
EXEMPLO

A RAZÃO ENTRE O ÂNGULO INTERNO E O ÂNGULO EXTERNO DE UM POLÍGONO REGULAR É 9.

DETERMINE O NÚMERO DE LADOS DESSE POLÍGONO.



$$\hat{a}_i + \hat{a}_e = 180^\circ$$

$$\frac{\hat{a}_i}{\hat{a}_e} = 9 \rightarrow \underline{\hat{a}_i = 9 \cdot \hat{a}_e}$$

$$9\hat{a}_e + \hat{a}_e = 180^\circ$$

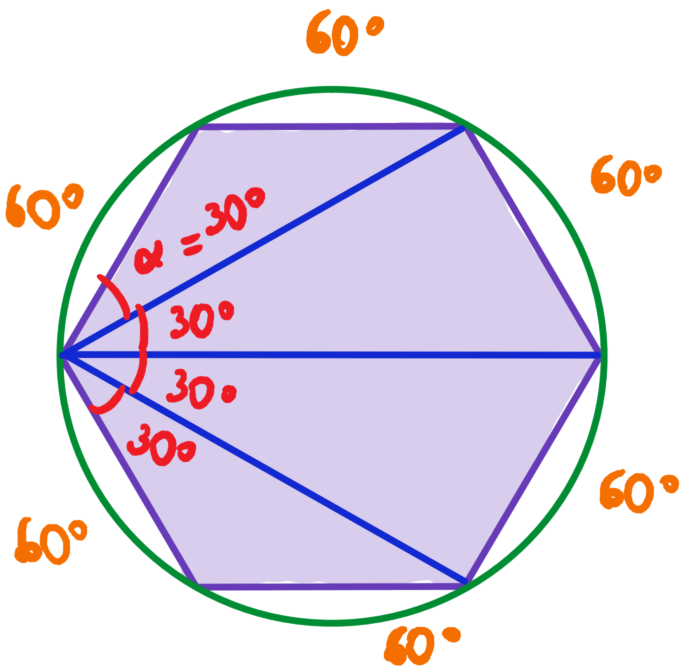
$$\underline{\hat{a}_e = 18^\circ}$$

$$\hat{a}_e = \frac{S_e}{n} \rightarrow 18^\circ = \frac{360^\circ}{n} \rightarrow n = \frac{360^\circ}{18^\circ}$$

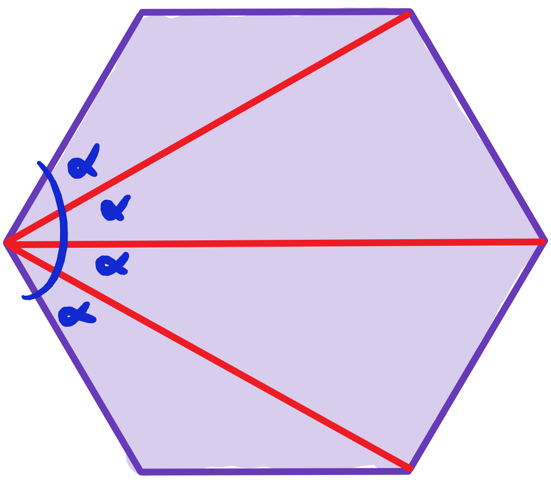
$$\underline{n = 20}$$

POLÍGONOS REGULARES E CIRCUNFERÊNCIAS

ÂNGULOS AO TRAÇAR AS DIAGONAIS:



$$\alpha = \frac{1}{2} \cdot \frac{360°}{n}$$



$$4\alpha = \hat{a}_i$$

$$4\alpha = 120°$$

$$\alpha = 30°$$

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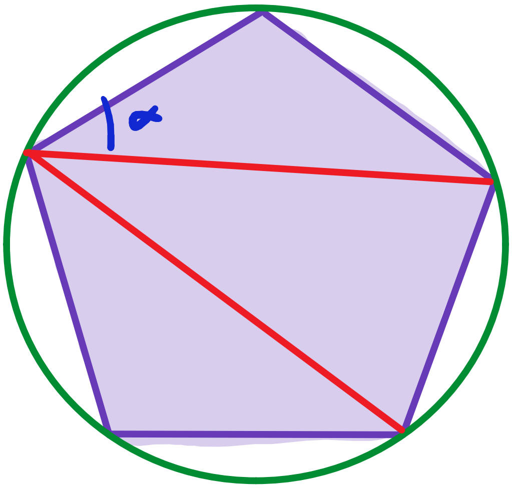
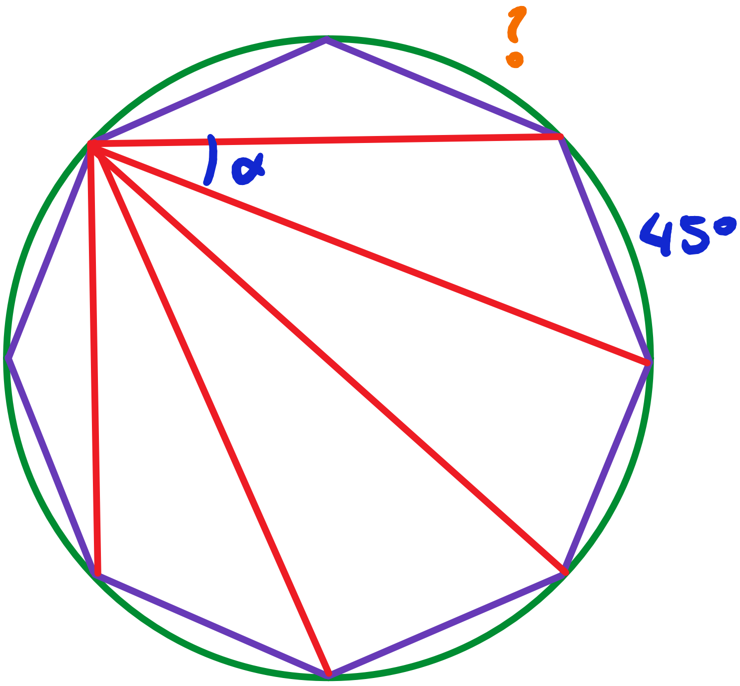


$$\alpha = \frac{a_i}{3}$$

$$\alpha = \frac{108}{3}$$

$$\underline{\alpha = 36^\circ}$$

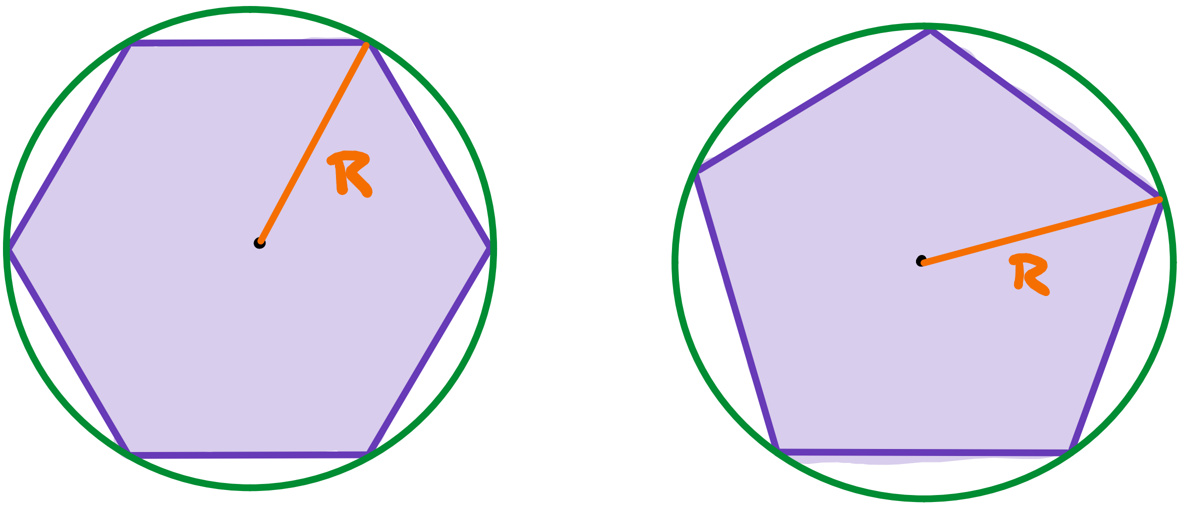


$$\frac{360}{8} = 45^\circ$$

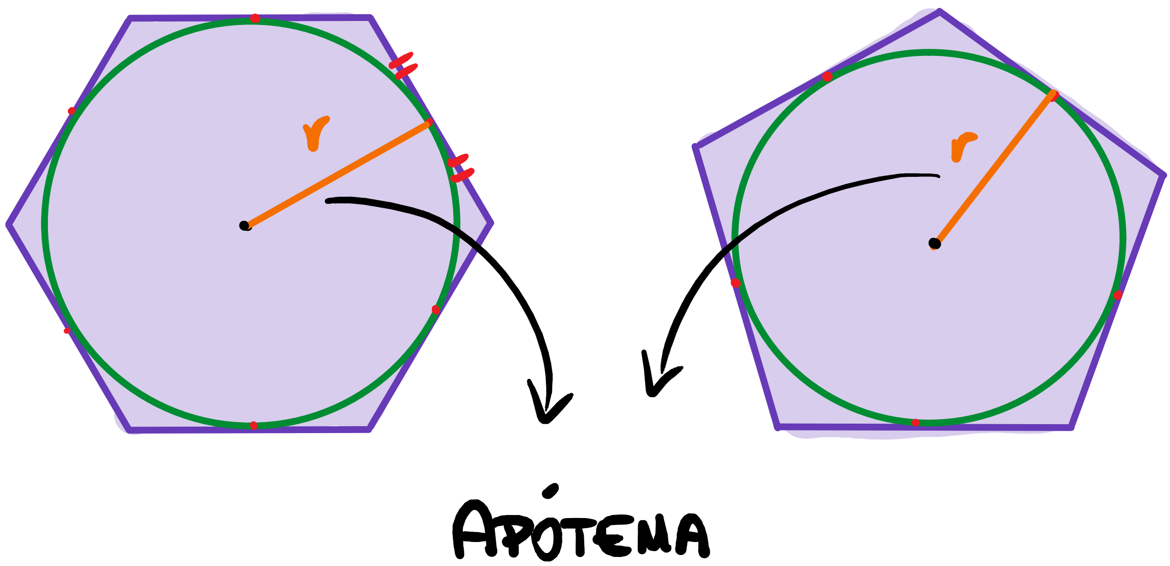
$$\alpha = \frac{45^\circ}{2}$$

$$\underline{\alpha = 22{,}5^\circ}$$

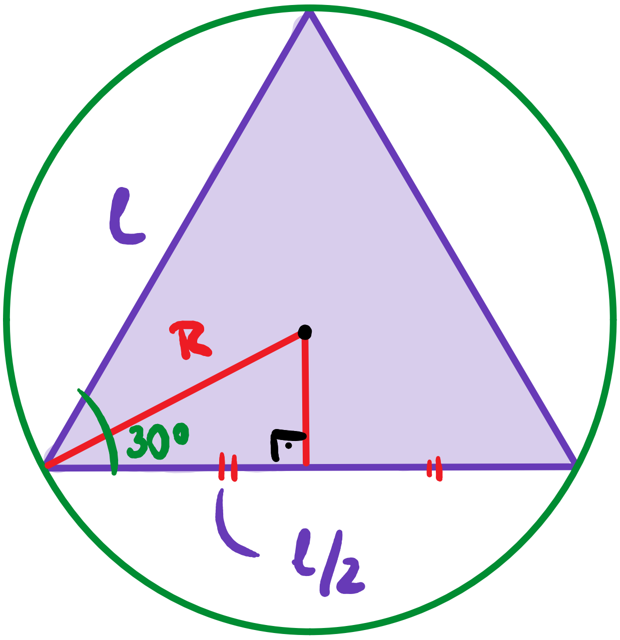
TODO POLÍGONO REGULAR É INSCRITÍVEL.



TODO POLÍGONO REGULAR É CIRCUNSCRITÍVEL.



TRIÂNGULO EQUILÁTERO

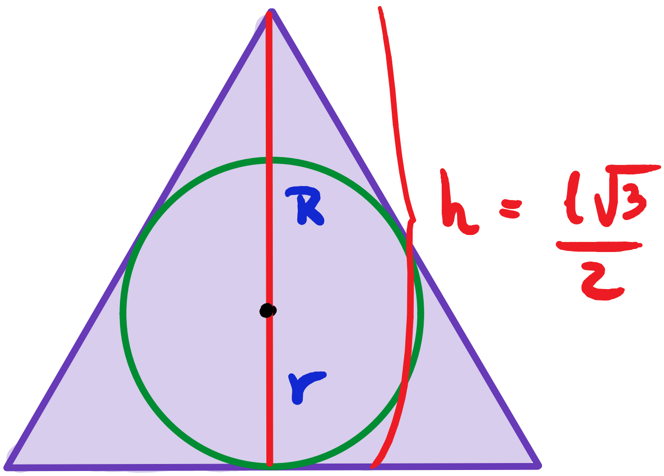


$$\cos 30° = \frac{\ell/2}{R}$$

$$R = \frac{\ell}{2} \cdot \frac{2}{\sqrt{3}}$$

$$R = \frac{\ell}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{R = \frac{\ell\sqrt{3}}{3}}$$

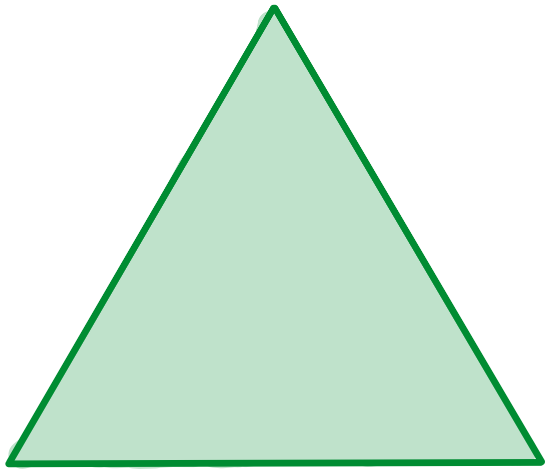


$$R + r = \frac{\ell\sqrt{3}}{2}$$

$$r = \frac{\ell\sqrt{3}}{2} - \frac{\ell\sqrt{3}}{3}$$

$$\boxed{r = \frac{\ell\sqrt{3}}{6}}$$

$$R = 2r$$

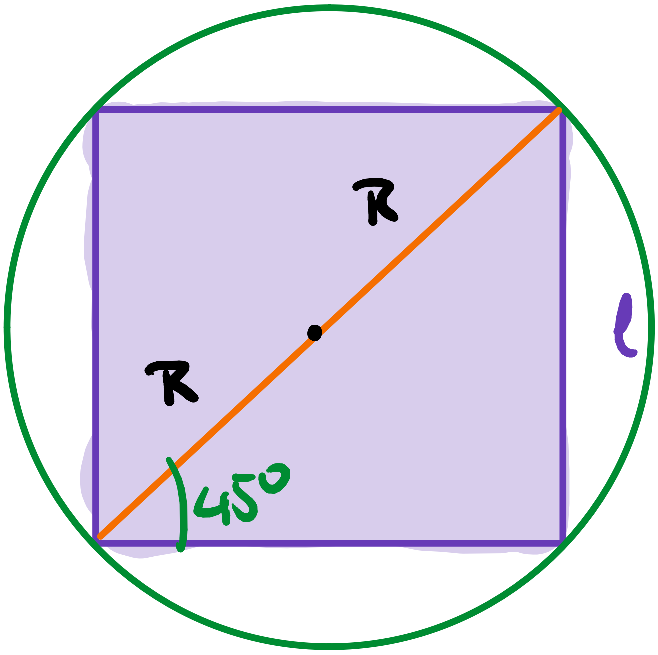


$$\ell = 7$$

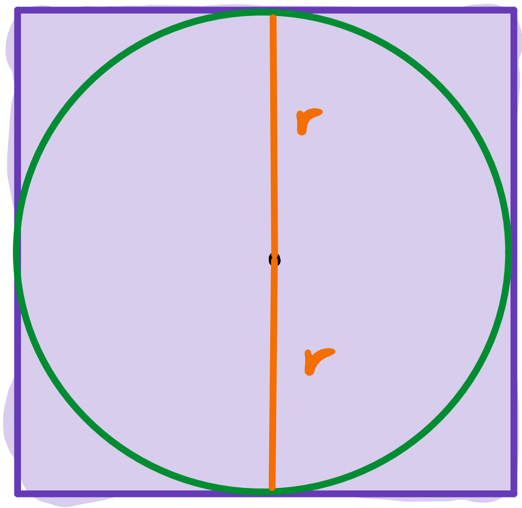
$$R = \frac{7\sqrt{3}}{3}$$

$$r = \frac{7\sqrt{3}}{6}$$

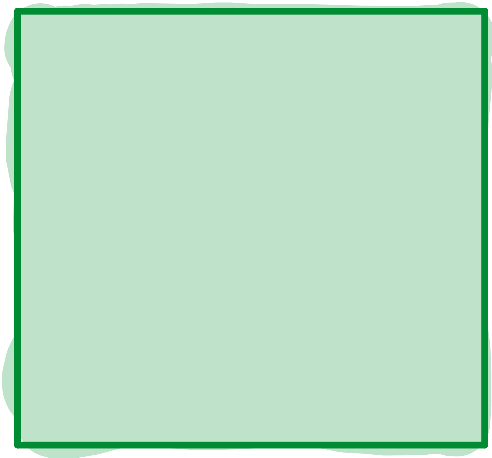
QUADRADO



$$\boxed{2R = l\sqrt{2}}$$



$$\boxed{2r = l}$$



$$R = 3$$

$$2 . 3 = \ell\sqrt{2}$$

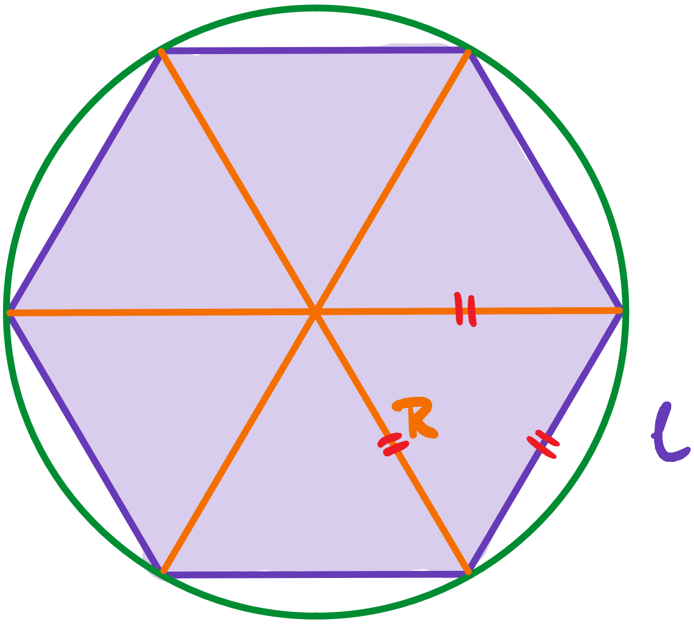
$$\ell = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\ell = \frac{6\sqrt{2}}{2}$$

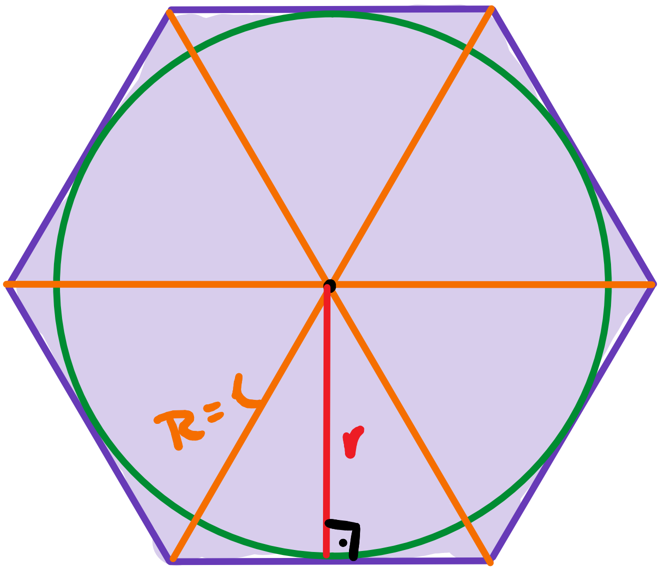
$$\underline{\ell = 3\sqrt{2}}$$

$$r = \frac{\ell}{2} \longrightarrow r = \frac{3\sqrt{2}}{2}$$

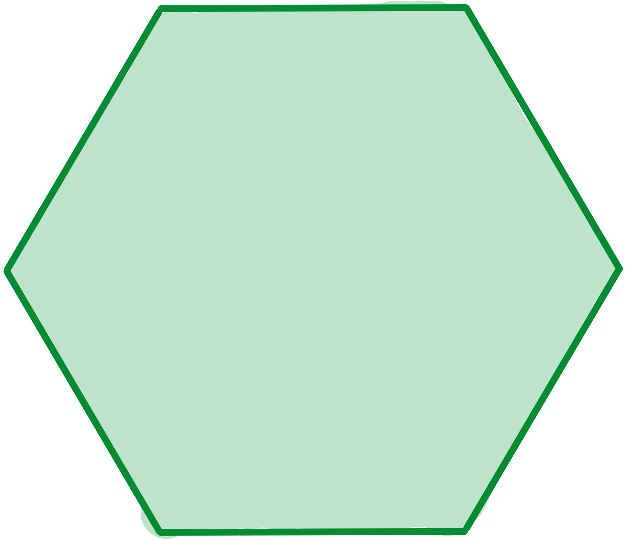
HEXÁGONO REGULAR



$$\boxed{R = l}$$



$$\boxed{r = \frac{R\sqrt{3}}{2}}$$



$$r = \sqrt{3}$$

$$\cancel{\sqrt{3}} = \frac{R\,\cancel{\sqrt{3}}}{2}$$

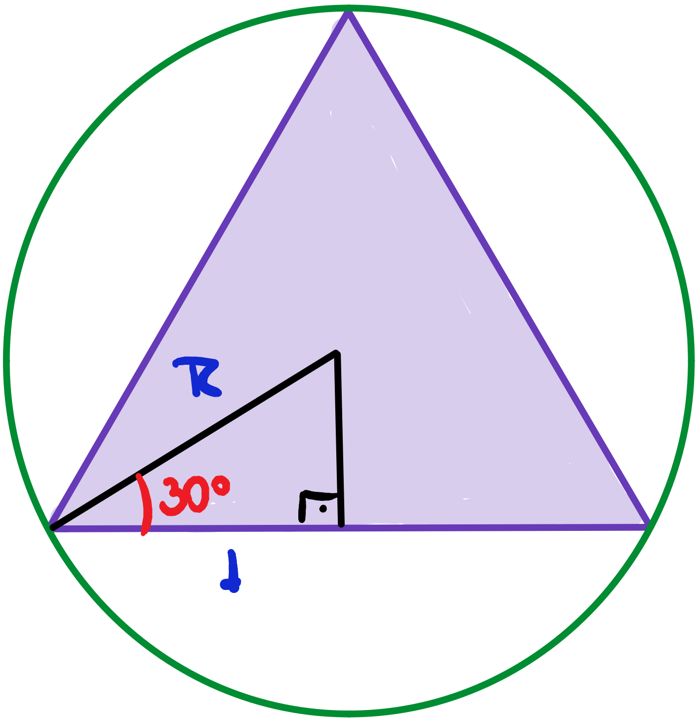
$$\underline{R = 2} \longrightarrow \underline{l = 2}$$

EXEMPLO

UMA MESA POSSUI A BASE NUM FORMATO TRIANGULAR REGULAR E O TAMPO (SUPERFÍCIE) NO FORMATO CIRCULAR.

SABENDO QUE O TAMPO DA MESA DEVE COBRIR POR INTEIRO A BASE, E QUE ESSA BASE POSSUI LADO 2m, CALCULE O RAIO MÍNIMO DESSE TAMPO.

$l = 2$



$$\cos 30° = \frac{1}{R}$$

$$R = \frac{1}{\cos 30}$$

$$R = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

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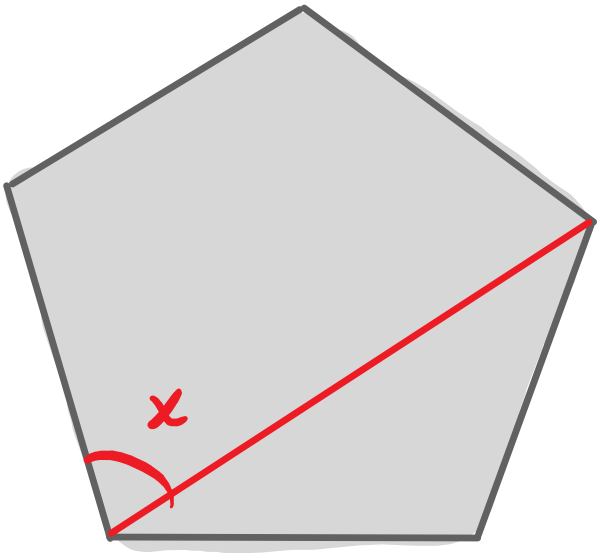
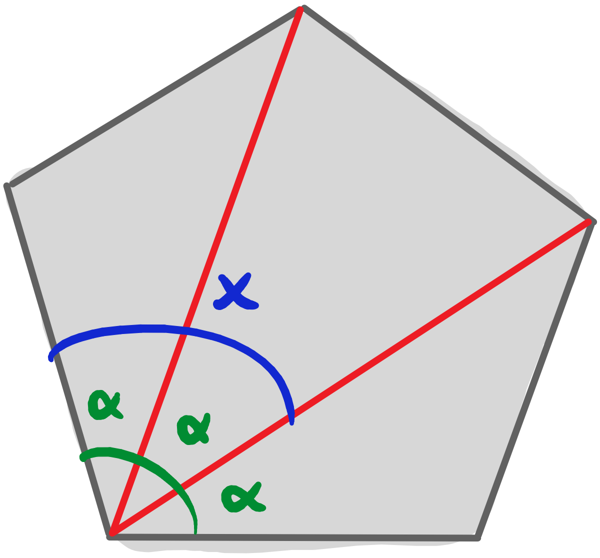
EXEMPLO

CALCULE O VALOR DO ÂNGULO x NO PENTÁGONO REGULAR ABAIXO.





$$3\alpha = \hat{a}_i$$

$$3\alpha = \frac{180(5-2)}{5}$$

$$3\alpha = 108^\circ$$

$$\underline{\alpha = 36^\circ}$$

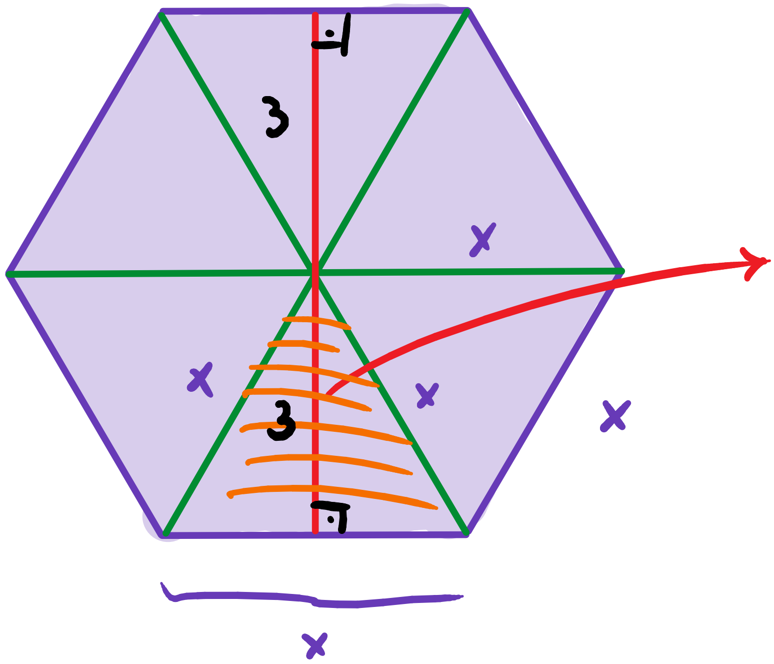
$$x = 2\alpha$$

$$\underline{x = 72^\circ}$$

EXEMPLO

A DISTÂNCIA ENTRE 2 LADOS PARALELOS DE UM HEXÁGONO REGULAR É 6.

CALCULE O PERÍMETRO DESSE HEXÁGONO.



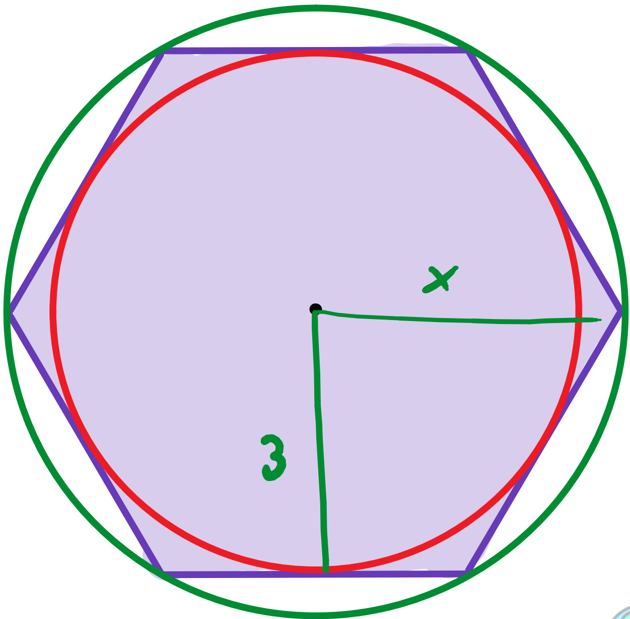
$$\frac{x\sqrt{3}}{2} = 3$$

$$x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{6\sqrt{3}}{3}$$

$$x = 2\sqrt{3}$$

$$2p = 6x = 12\sqrt{3}$$

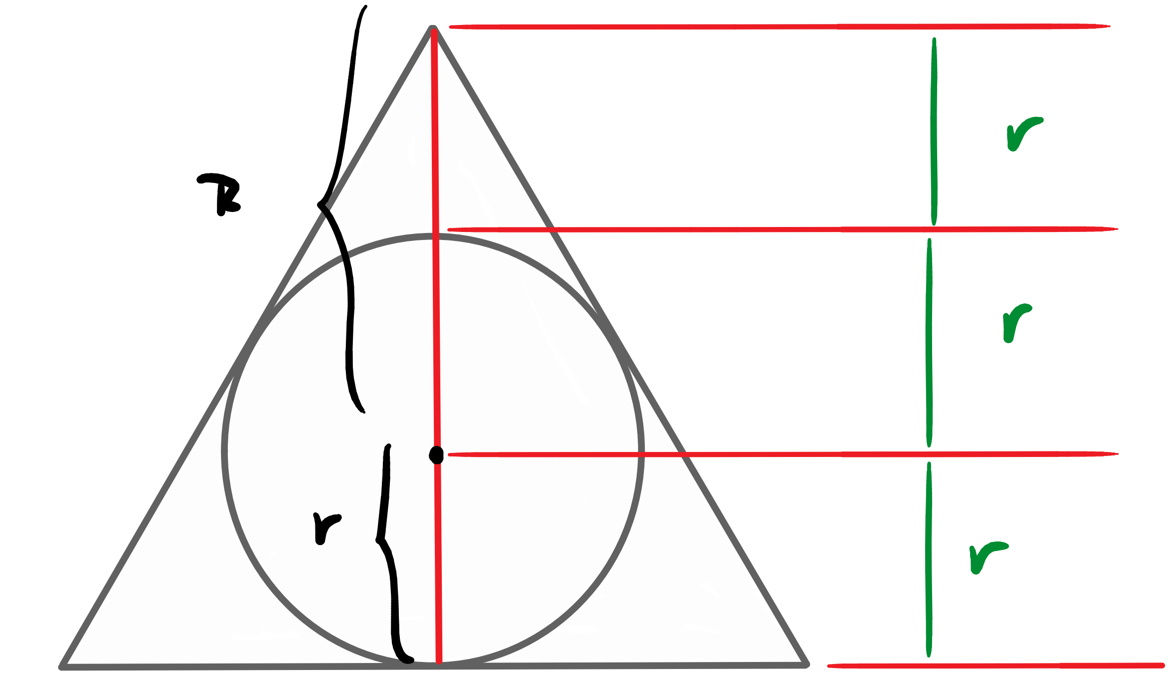


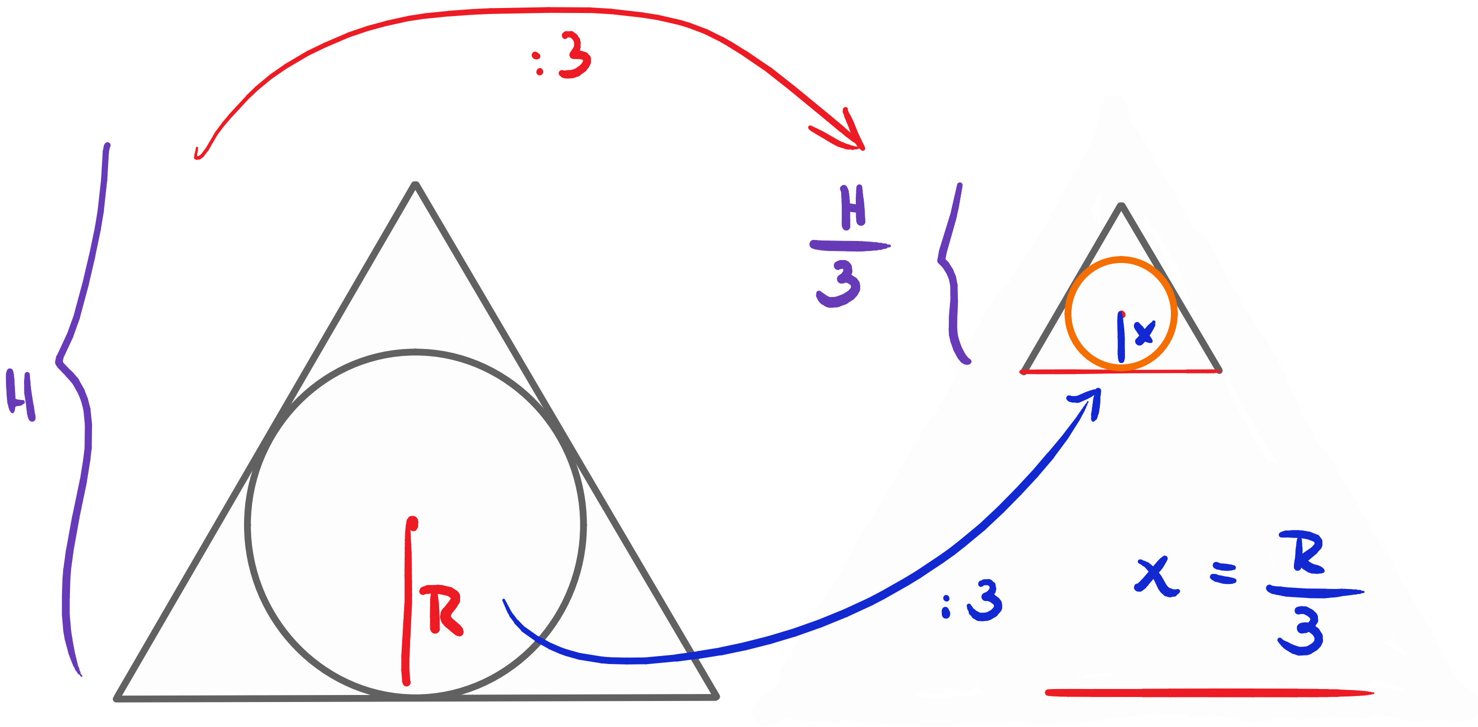
EXEMPLO

UM TRIÂNGULO EQUILÁTERO ESTÁ CIRCUNSCRITO A UMA CIRCUNFERÊNCIA DE RAIO R.

CALCULE O RAIO DA CIRCUNFERÊNCIA TANGENTE A ESSA CIRCUNFERÊNCIA E TAMBÉM A DOIS LADOS DO TRIÂNGULO.

$R = 2r$



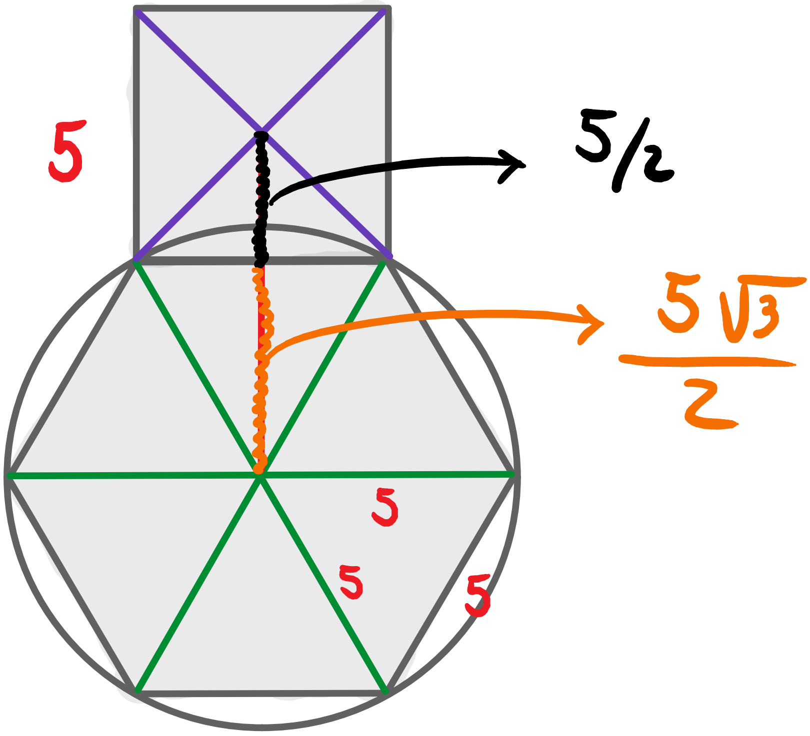


$x = \frac{R}{3}$

EXEMPLO

UM HEXÁGONO REGULAR ESTÁ INSCRITO EM UM CÍRCULO DE RAIO 5. UM DOS LADOS DO HEXÁGONO TAMBÉM É LADO DE UM QUADRADO CONSTRUÍDO EXTERIORMENTE AO HEXÁGONO.

CALCULE A DISTÂNCIA ENTRE O CENTRO DO CÍRCULO E A INTERSEÇÃO DAS DIAGONAIS DO QUADRADO.



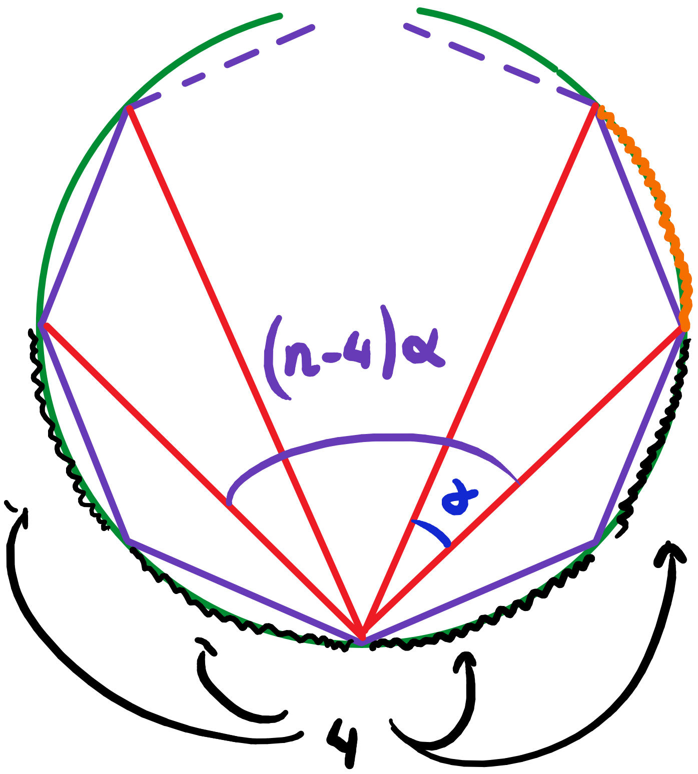
$$d = \frac{5\sqrt{3}}{2} + \frac{5}{2}$$

$$d = \frac{5}{2}(\sqrt{3} + 1)$$

EXEMPLO

NUM POLÍGONO REGULAR FORAM TRAÇADAS TODAS AS DIAGONAIS QUE PARTEM DE UM DE SEUS VÉRTICES. O ÂNGULO FORMADO ENTRE A PRIMEIRA E A ÚLTIMA DIAGONAL É IGUAL AO ÂNGULO EXTERNO DESSE POLÍGONO.

CALCULE O NÚMERO DE DIAGONAIS TRAÇADAS.



$$\frac{360^\circ}{n}$$

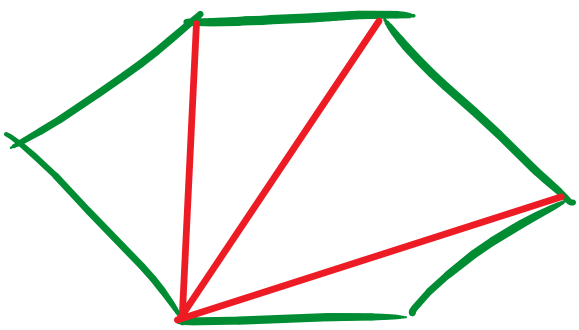
$$\alpha = \frac{1}{2} \cdot \frac{360}{n}$$

$$\alpha = \frac{180^\circ}{n}$$

$$(n-4) \cdot \frac{\cancel{180^\circ}}{\cancel{n}} = \frac{\cancelto{2}{360^\circ}}{\cancel{n}}$$

$$n-4 = 2$$

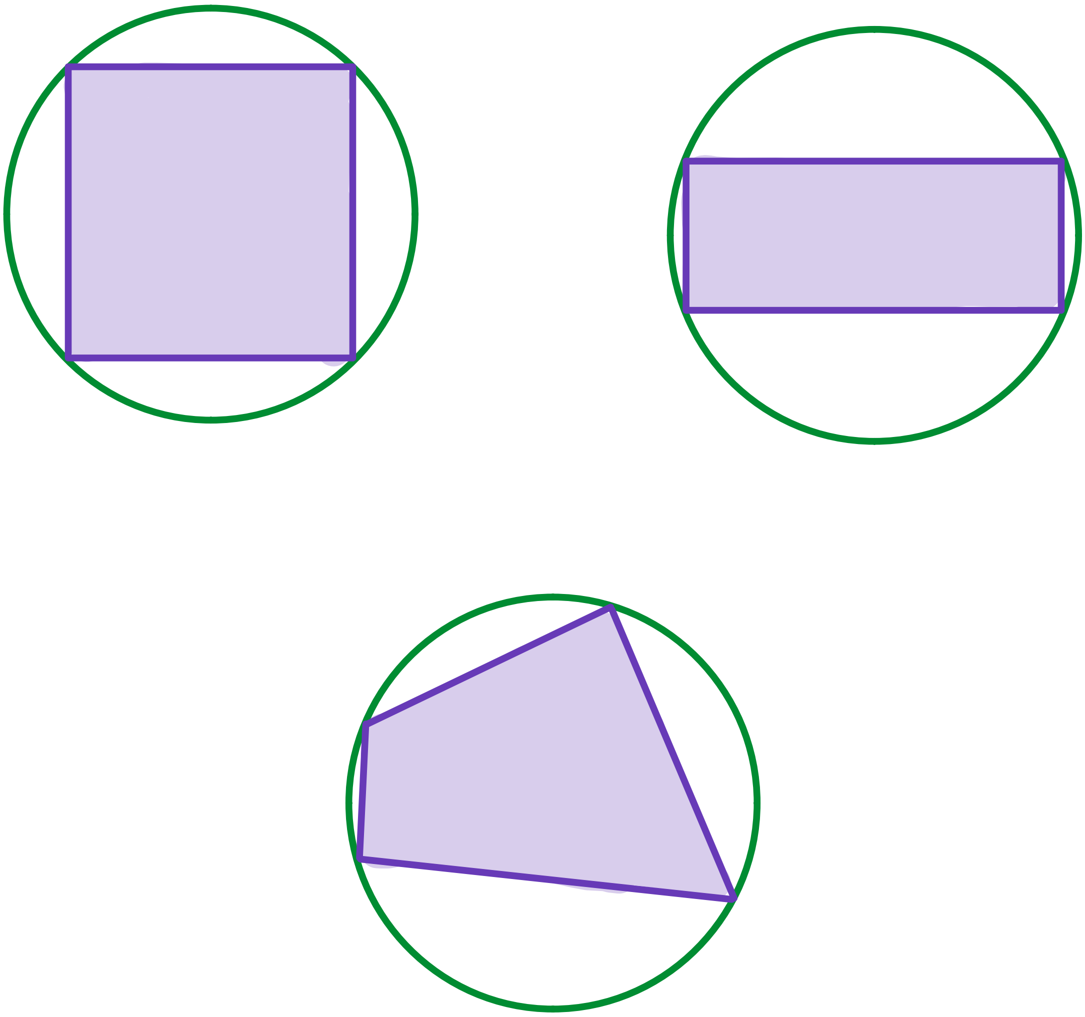
$$n = 6$$



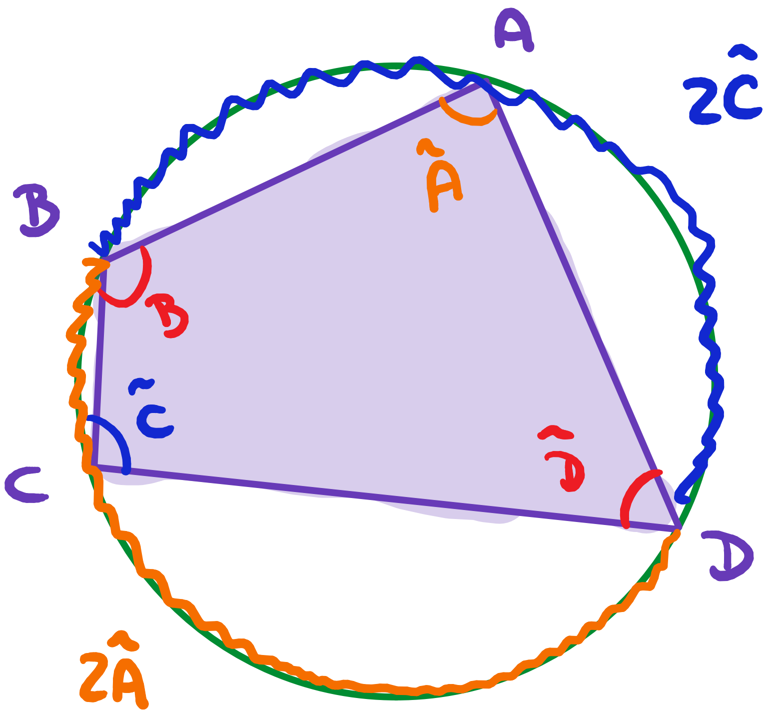
3 DIAGONAIS

QUADRILÁTEROS INCRITÍVEIS

SÃO QUADRILÁTEROS QUE PODEM SER INSCRITOS EM UMA CIRCUNFERÊNCIA.

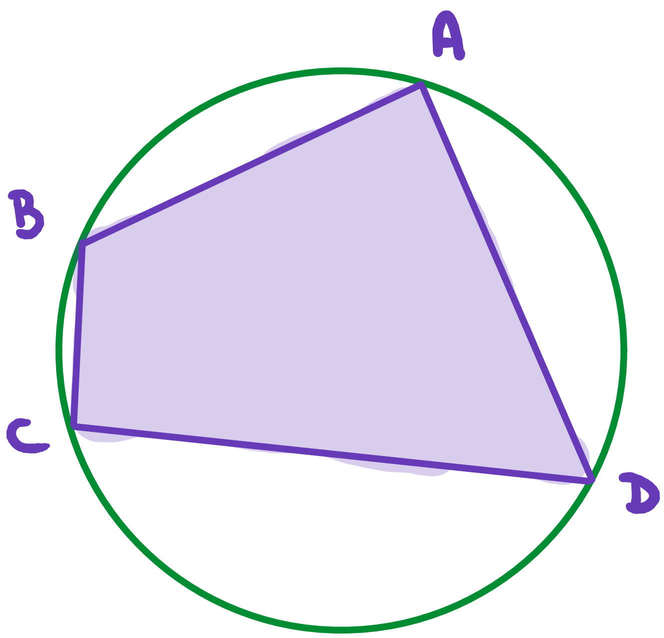


CONDIÇÃO:



$$2\hat{A} + 2\hat{C} = 360°$$

$$\hat{A} + \hat{C} = 180°$$

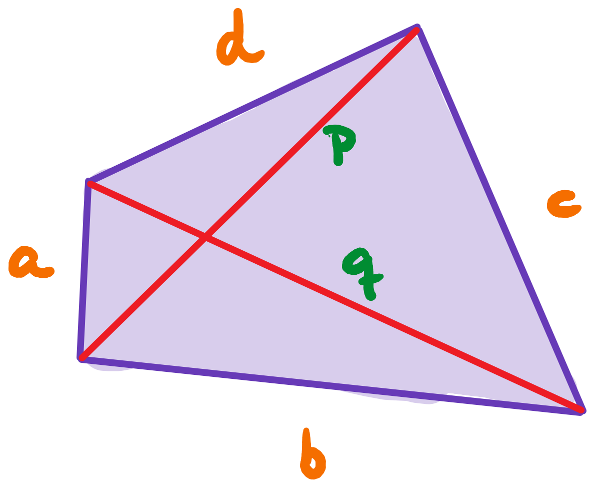


$$\hat{A} + \hat{C} = 180°$$

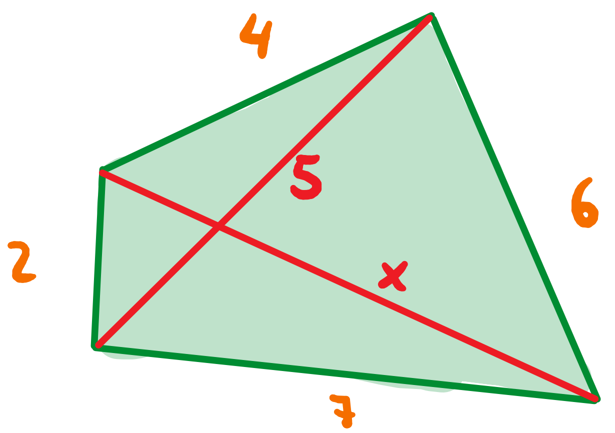
$$\hat{B} + \hat{D} = 180°$$

TEOREMA DE PTOLOMEU

SEJA O QUADRILÁTERO ABCD INCRITÍVEL.



$$\boxed{p \cdot q = a \cdot c + b \cdot d}$$



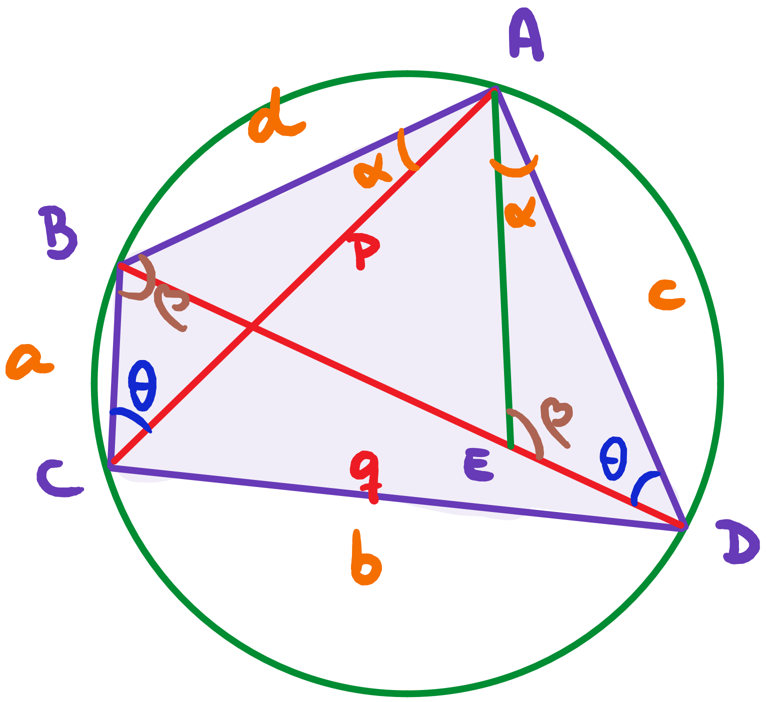
$$5 \cdot x = 2 \cdot 6 + 7 \cdot 4$$

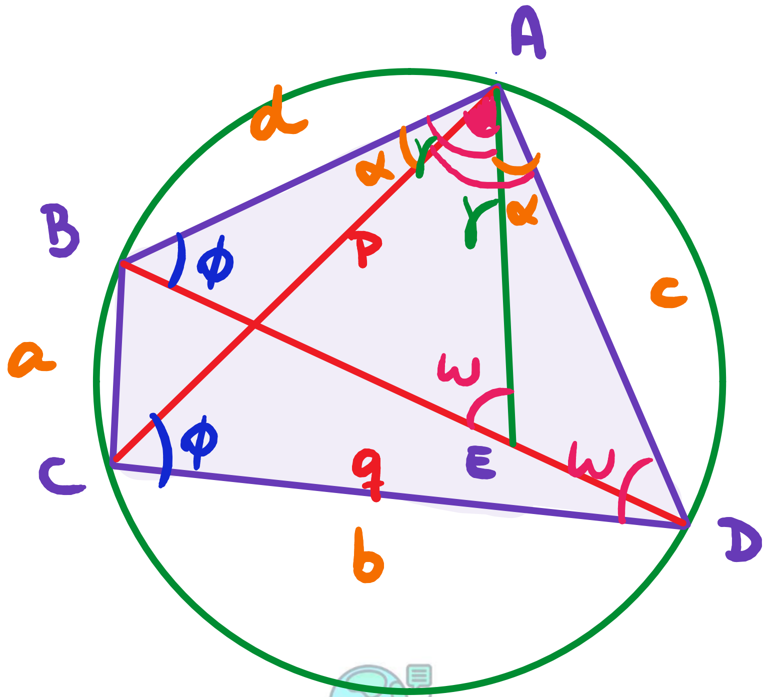
$$5x = 40$$

$$x = 8$$

TEOREMA DE PTOLOMEU

DEMONSTRAÇÃO





$\Delta ABC \sim \Delta AED$

$$\frac{a}{ED} = \frac{p}{c} \rightarrow \boxed{ED = \frac{a.c}{p}}$$

$\Delta ABE \sim \Delta ACD$

$$\frac{BE}{b} = \frac{d}{p} \rightarrow \boxed{BE = \frac{bd}{p}}$$

$$ED + BE = q$$

$$\frac{a.c}{p} + \frac{bd}{p} = q$$

$$\frac{ac + bd}{p} = q$$

$$\boxed{a.c + b.d = p.q}$$

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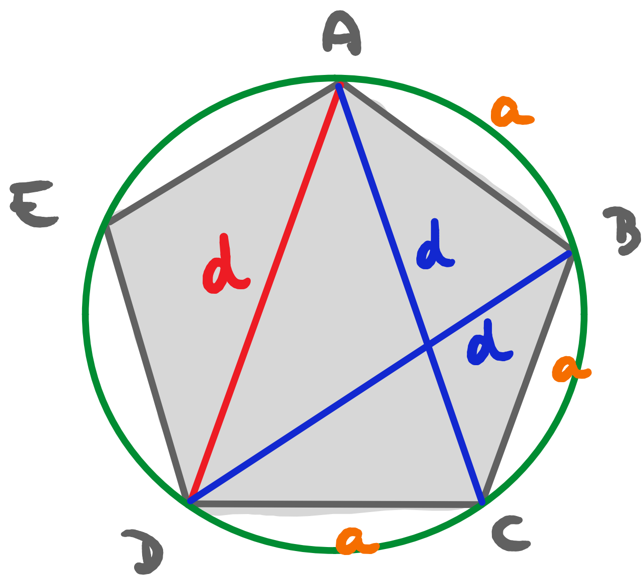
EXEMPLO

CALCULE A RAZÃO ENTRE A DIAGONAL E O LADO DE UM PENTÁGONO REGULAR.



$$d.d = a.d + a.a$$

$$\frac{d^2}{a^2} = \frac{ad}{a^2} + \frac{a^2}{a^2}$$

$$\left(\frac{d}{a}\right)^2 = \left(\frac{d}{a}\right) + 1$$

$$\frac{d}{a} = x \rightarrow x^2 - x - 1 = 0$$

$$\Delta = (-1)^2 - 4.1(-1) = 5$$

$$\frac{d}{a} = \frac{1 \pm \sqrt{5}}{2} \qquad \boxed{\frac{d}{a} = \frac{1 + \sqrt{5}}{2}}$$

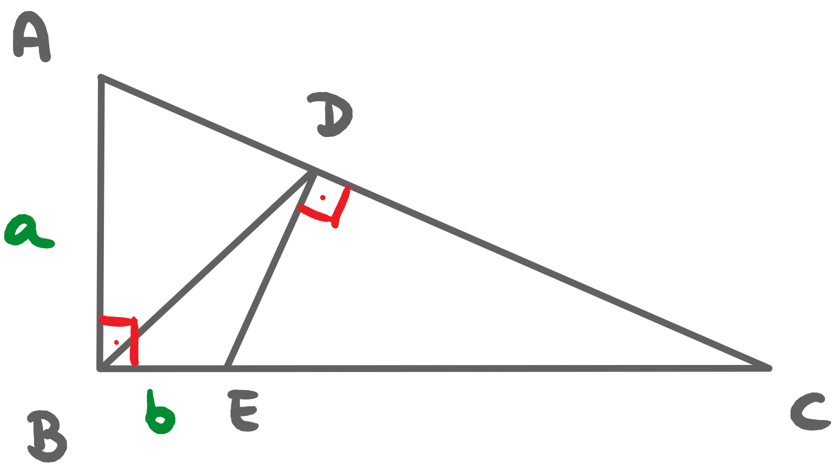
Nº DE OURO

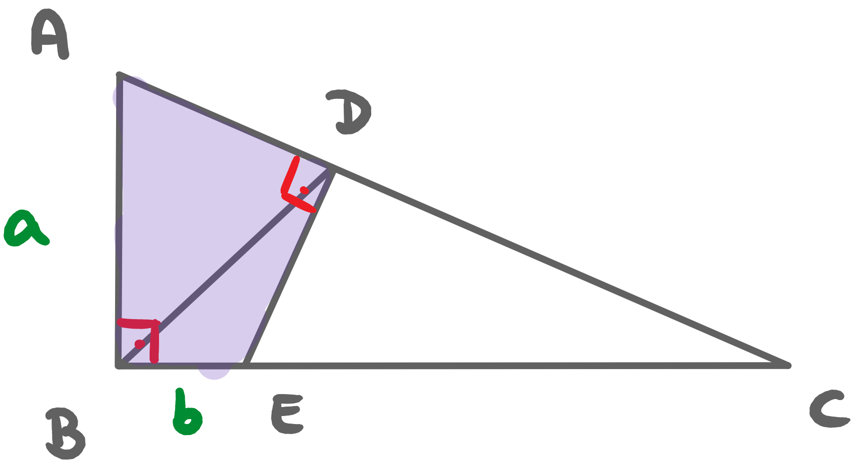
$\phi \sim 1{,}618$

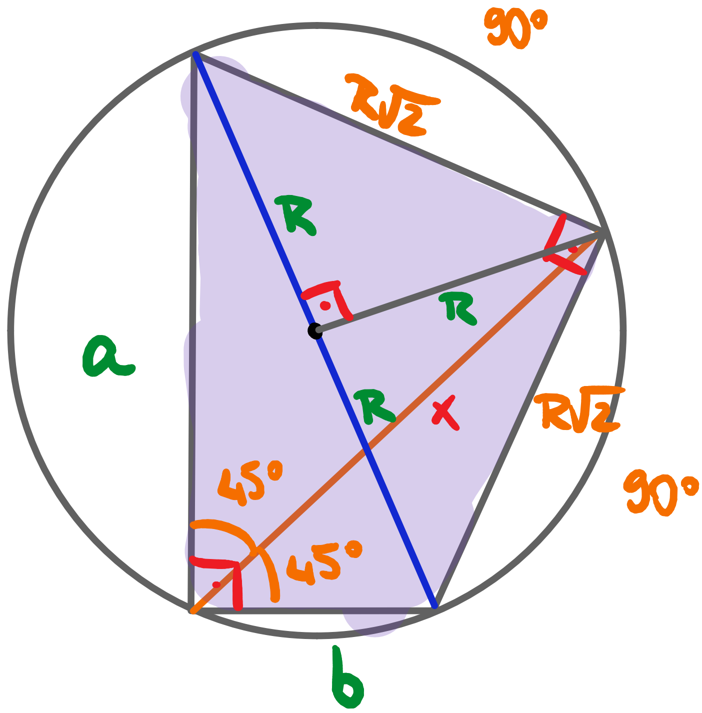
EXEMPLO

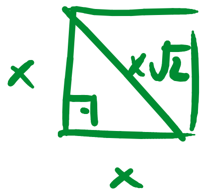
NA FIGURA ABAIXO, BD É BISSETRIZ DO ÂNGULO B DO TRIÂNGULO ABC, RETÂNGULO EM B. TRAÇA-SE ED DE MODO QUE O ÂNGULO $A\hat{D}E$ SEJA RETO.

SABENDO QUE $AB = a$ E $BE = b$, CALCULE O COMPRIMENTO DE BD.









$$x \cdot 2R = a \cdot R\sqrt{2} + b \cdot R\sqrt{2}$$

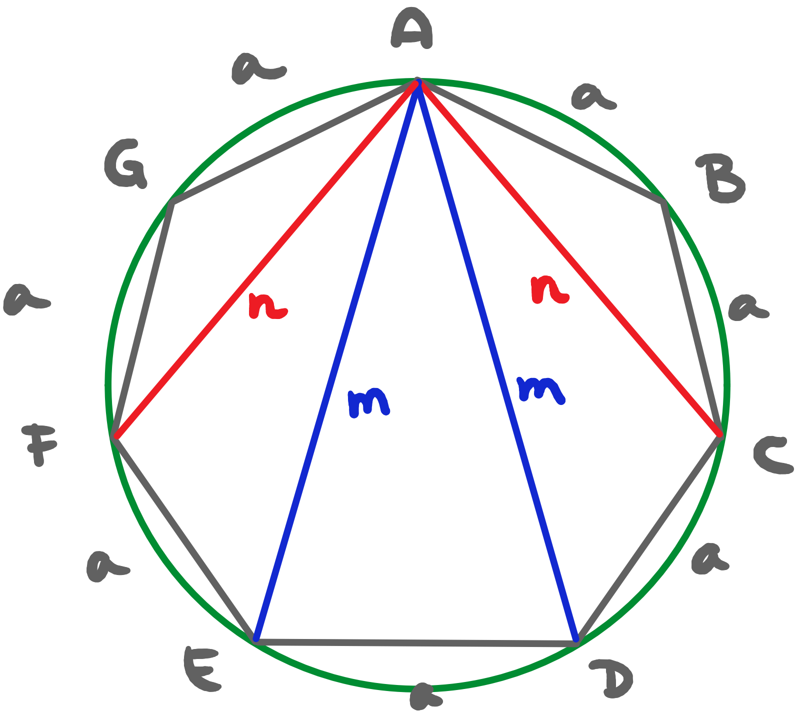
$$2x\cancel{R} = \cancel{R}\sqrt{2}(a+b)$$

$$\boxed{x = \frac{(a+b)\sqrt{2}}{2}}$$

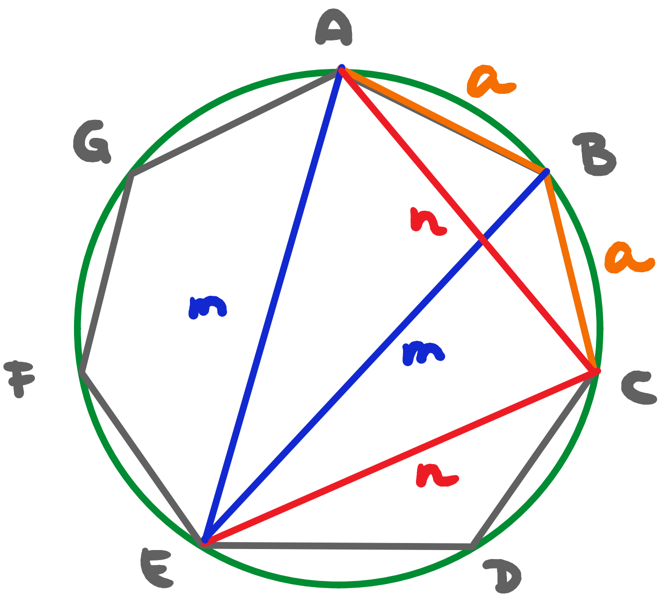
EXEMPLO

SEJA O HEPTÁGONO REGULAR ABCDEFG.

MOSTRE QUE $1/AB = 1/AC + 1/AD$



$$\frac{1}{a} = \frac{1}{n} + \frac{1}{m}$$



"ABCE"

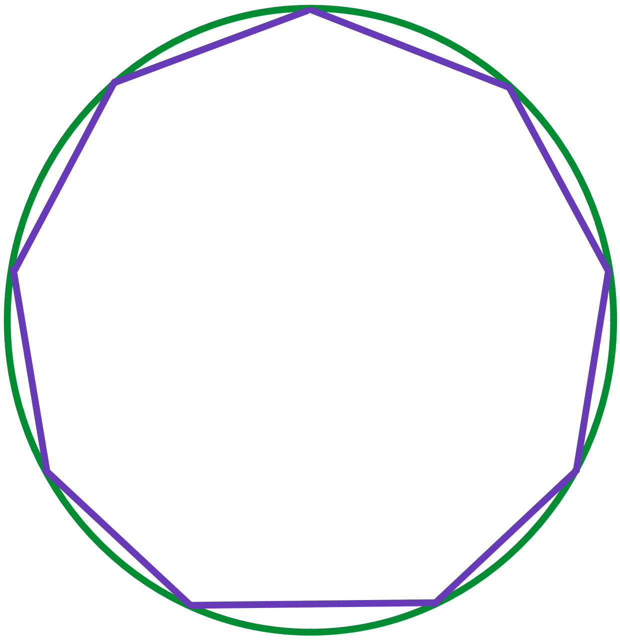
$$\frac{m.n}{amn} = \frac{a.m}{amn} + \frac{a.n}{amn}$$

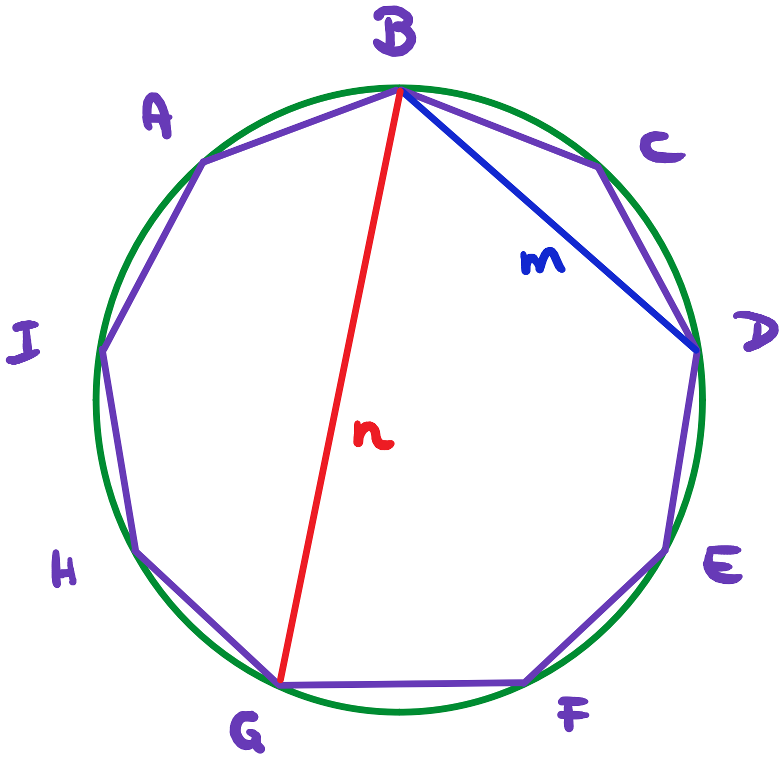
$$\boxed{\frac{1}{a} = \frac{1}{m} + \frac{1}{n}}$$

EXEMPLO

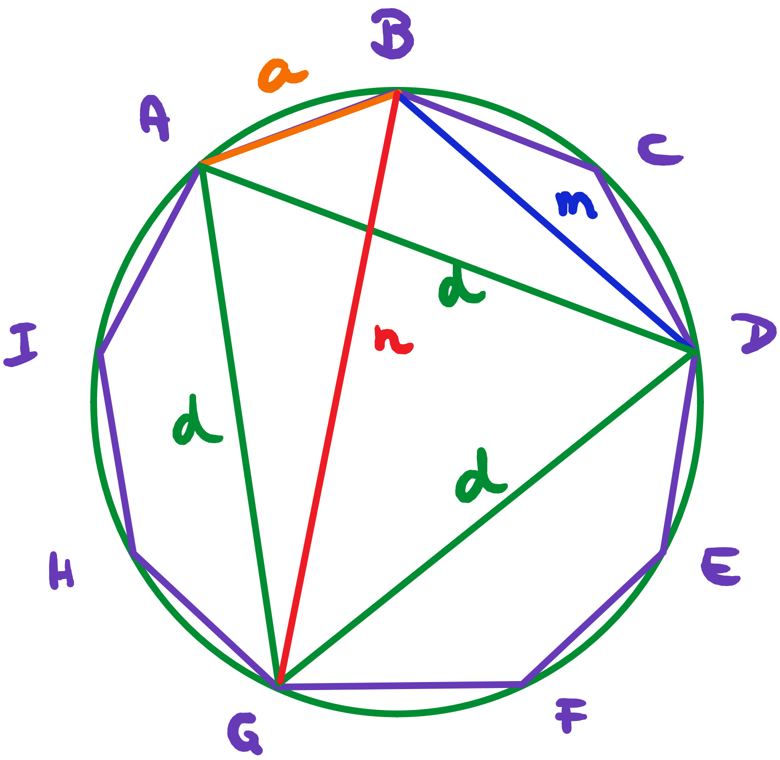
CONSIDERE UM ENEÁGONO ABCD... TAL QUE $BG - BD = 11$.

CALCULE O PERÍMETRO DESSE ENEÁGONO.





$n - m = 11$



ABDG

$n \cdot \cancel{d} = a \cdot \cancel{d} + m \cdot \cancel{d}$

$\underline{a = n - m}$

$a = 11$

$PER = 9 \cdot 11$

$\underline{PER = 99}$