SEMELHANÇA DE TRIÂNGULOS

FIGURAS SEMELHANTES

SÃO FIGURAS QUE ESTÃO NA MESMA PROPORÇÃO.

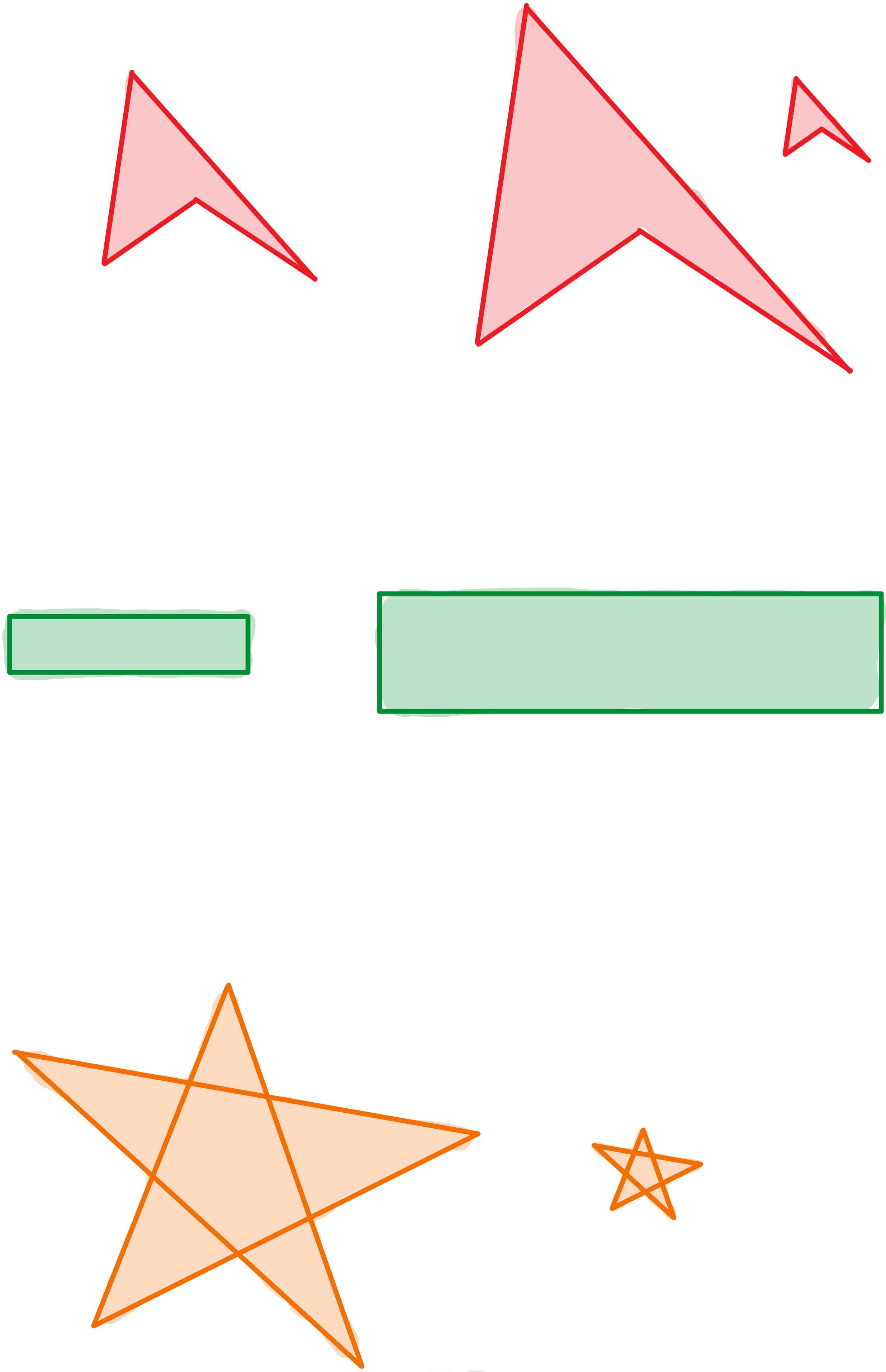
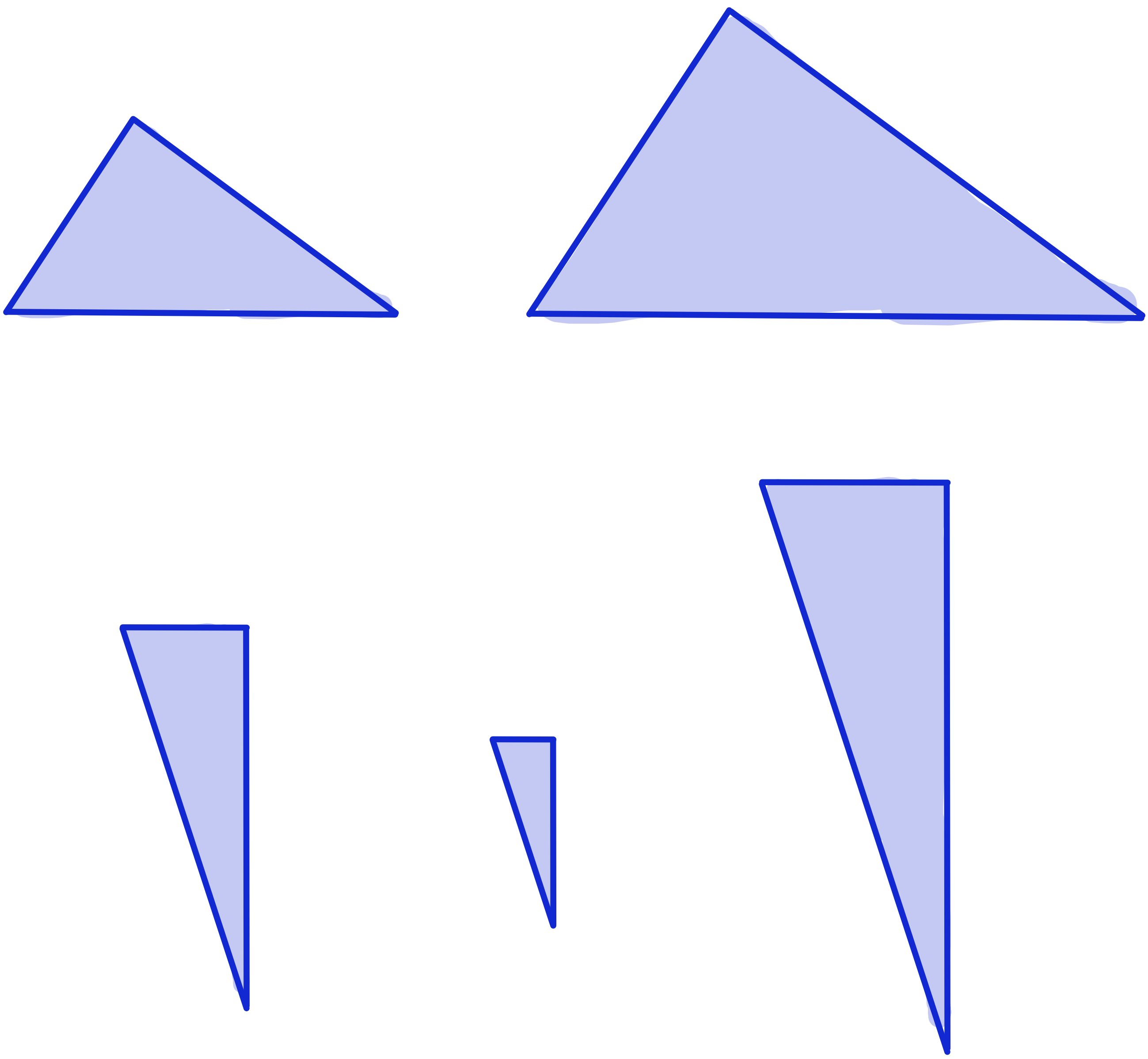


FIGURA MAIS IMPORTANTE NO ESTUDO DA SEMELHANÇA:

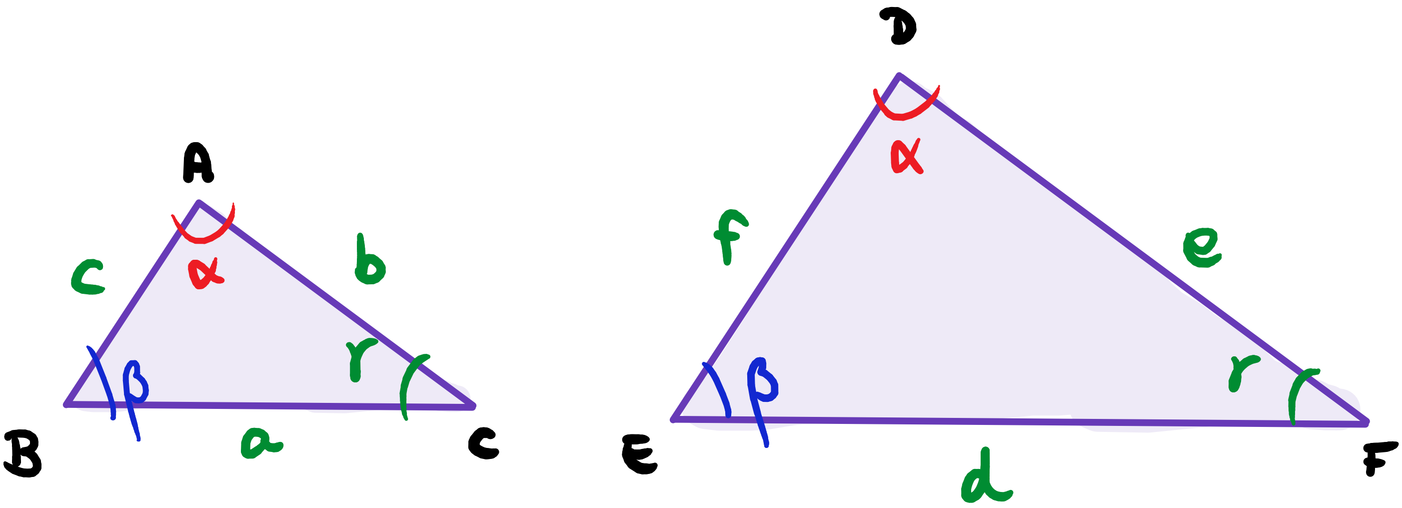
TRIÂNGULO



TRIÂNGULOS SEMELHANTES

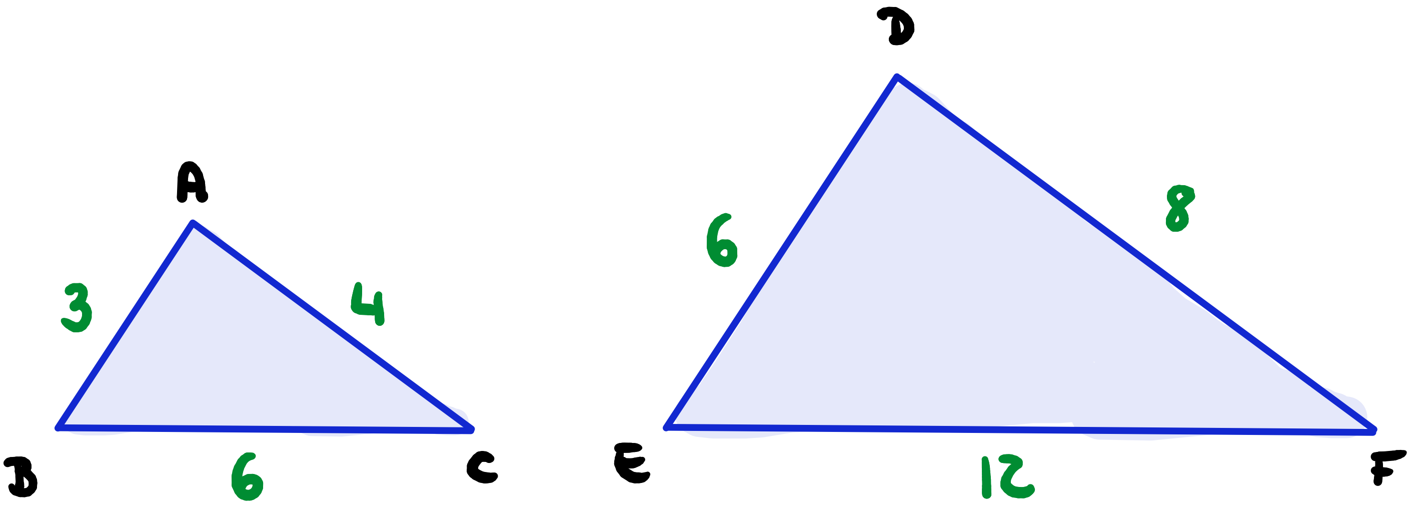
TRIÂNGULOS SÃO SEMELHANTES QUANDO:

- ✓ ÂNGULOS OS SÃO CONGRUENTES;
- ✓ LADOS CORRESPONDENTES PROPORCIONAIS.

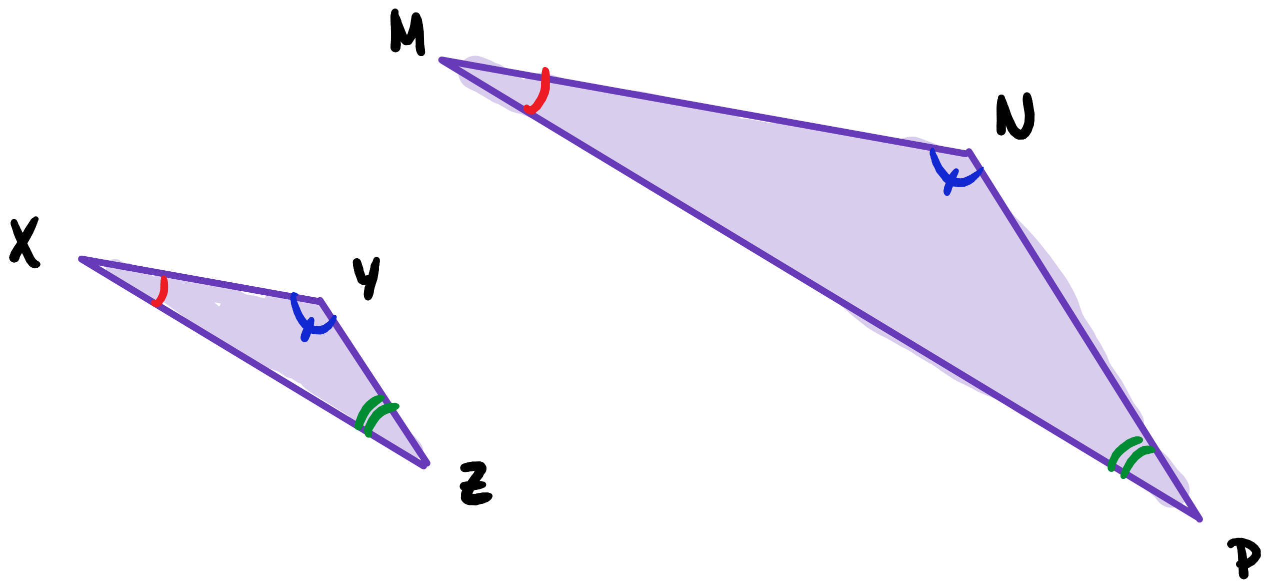


$$\boxed{\frac{a}{d} = \frac{b}{e} = \frac{c}{f}}$$

$$\Delta ABC \sim \Delta DEF$$



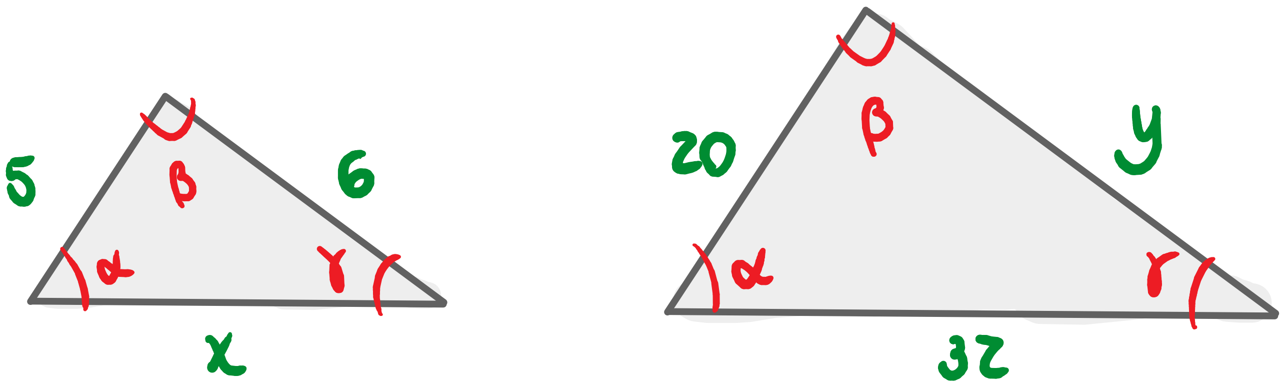
$$\frac{6}{3} = \frac{8}{4} = \frac{12}{6}$$



$$\hat{X} = \hat{M} \qquad \hat{Y} = \hat{N} \qquad \hat{Z} = \hat{P}$$

EXEMPLO

DETERMINE OS VALORES DAS INCÓGNITAS:



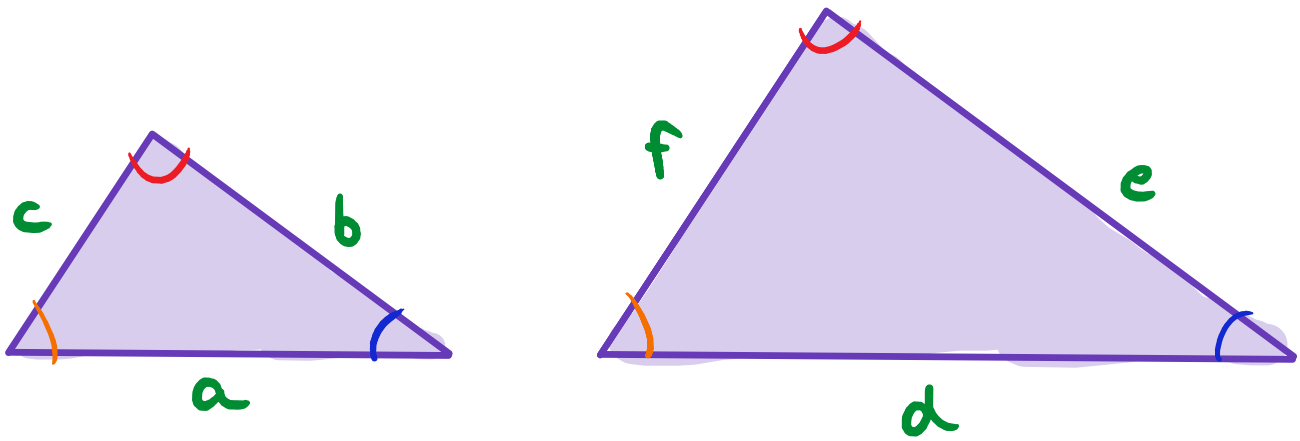
$$\frac{y}{6} = \frac{32}{x} = \frac{20}{5}$$

$$\frac{y}{6} = \frac{\cancel{20}^{\,4}}{\cancel{5}_{\,1}} \rightarrow \underline{y = 24}$$

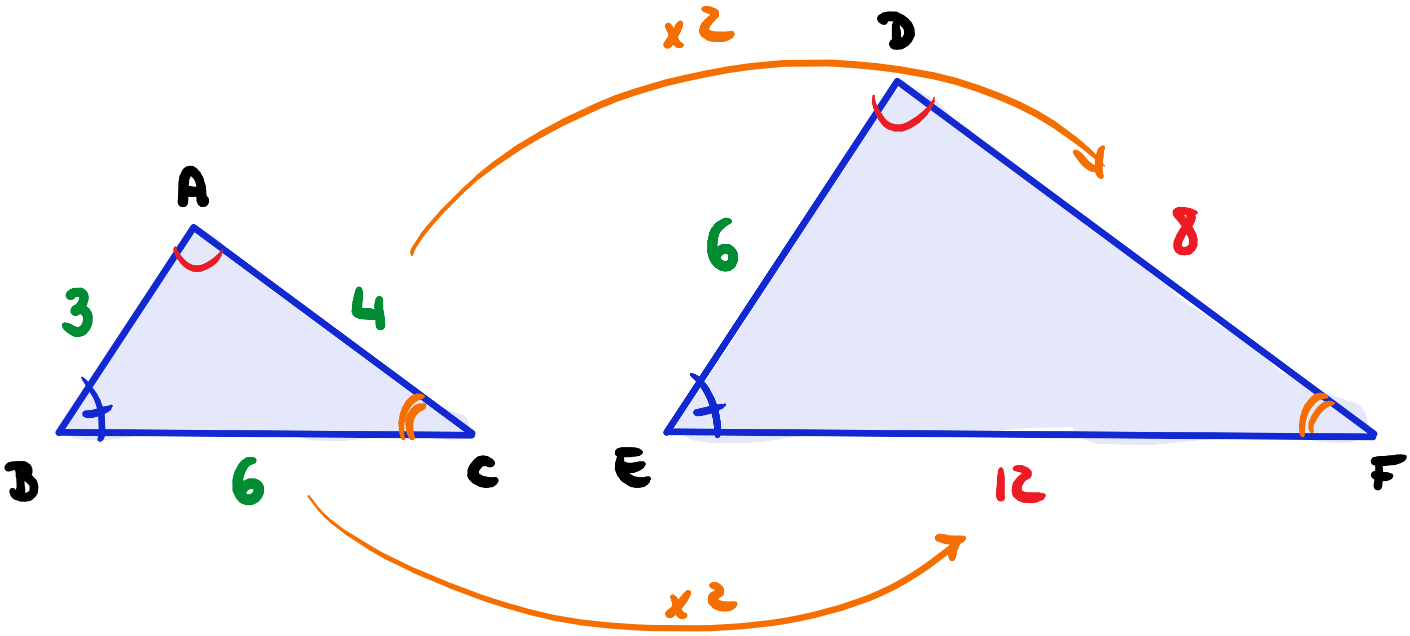
$$\frac{32}{x} = \frac{\cancel{20}^{\,4}}{\cancel{5}_{\,1}} \rightarrow 4x = 32 \rightarrow \underline{x = 8}$$

RAZÃO DE SEMELHANÇA (K)

RAZÃO ENTRE MEDIDAS CORRESPONDENTES



$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = K$$



$$K = \frac{6}{3}$$

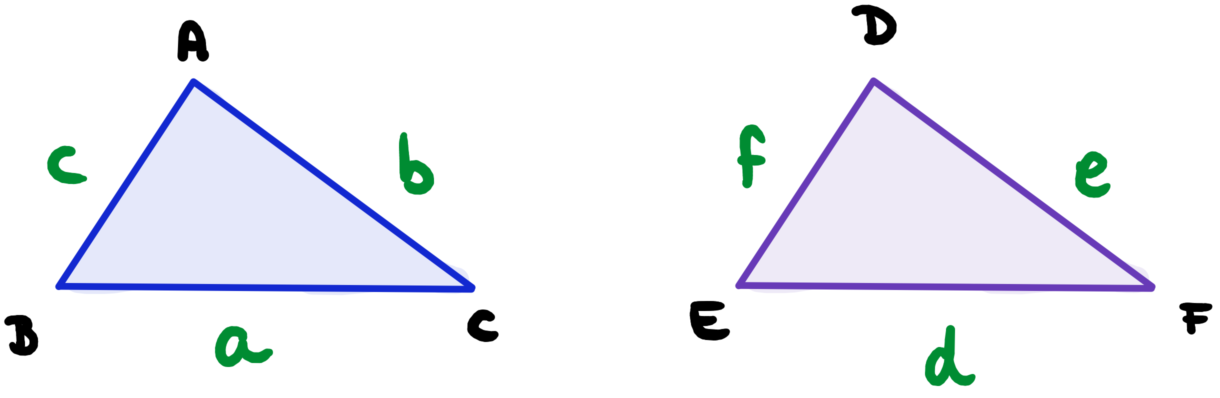
$$\underline{K = 2}$$

CONGRUÊNCIA DE TRIÂNGULOS

TRIÂNGULOS SEMELHANTES COM:

$$\boxed{K = 1}$$

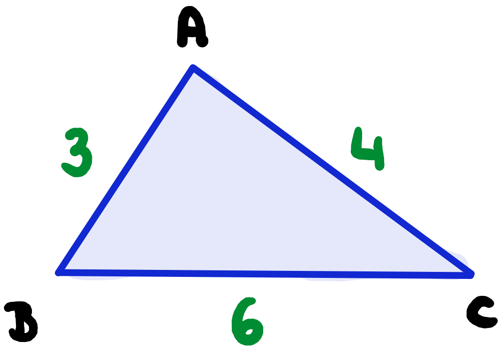
"TRIÂNGULOS IGUAIS"

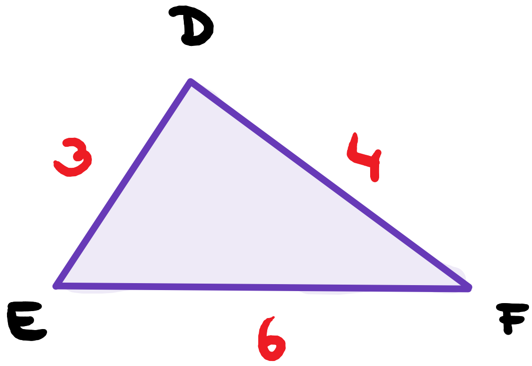


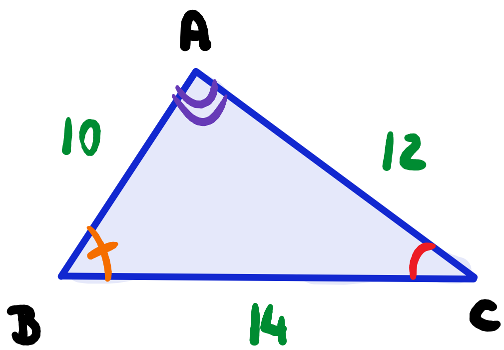
$$\frac{a}{d} = 1 \rightarrow a = d$$

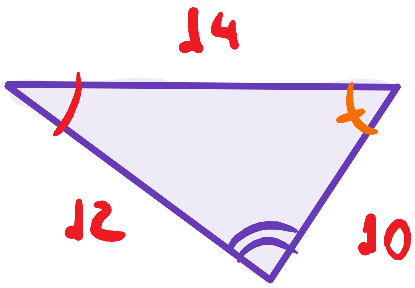
$$\frac{b}{e} = 1 \rightarrow b = e$$

$$\frac{c}{f} = 1 \rightarrow c = f$$









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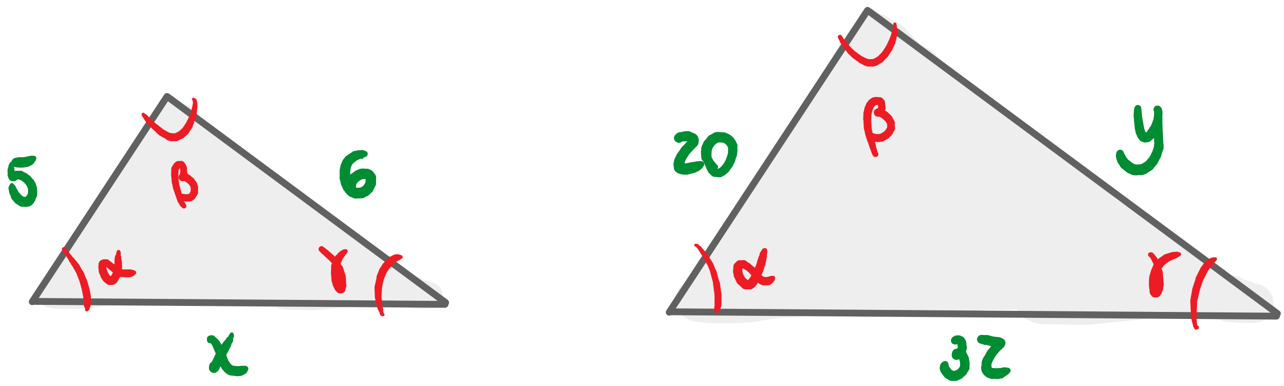
EXEMPLO

DETERMINE OS VALORES DAS INCÓGNITAS



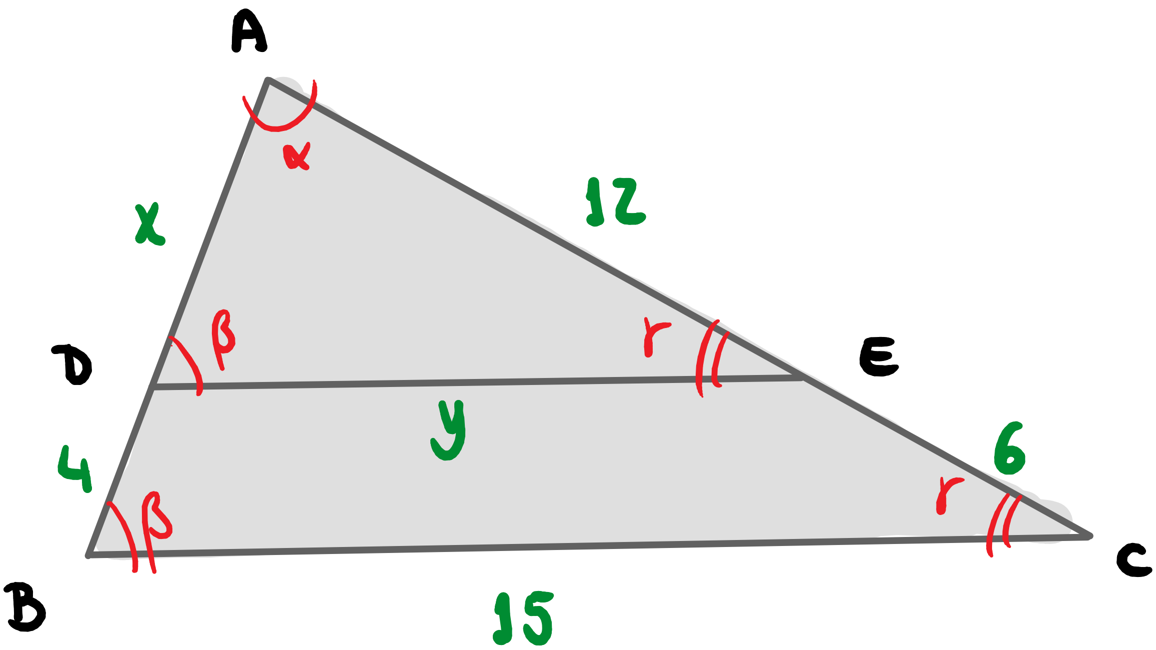
$K = \frac{20}{5} \rightarrow \underline{K = 4}$

$y = 4 \cdot 6 \rightarrow \underline{y = 24}$

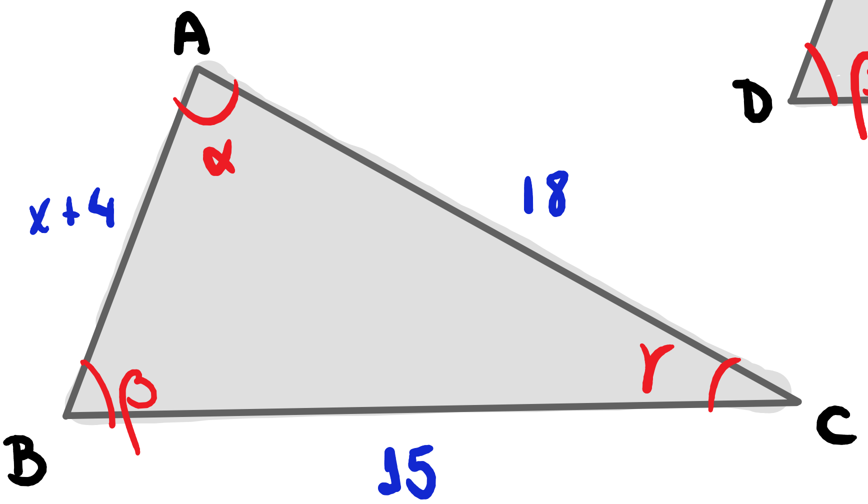
$32 = 4 \cdot x \rightarrow \underline{x = 8}$

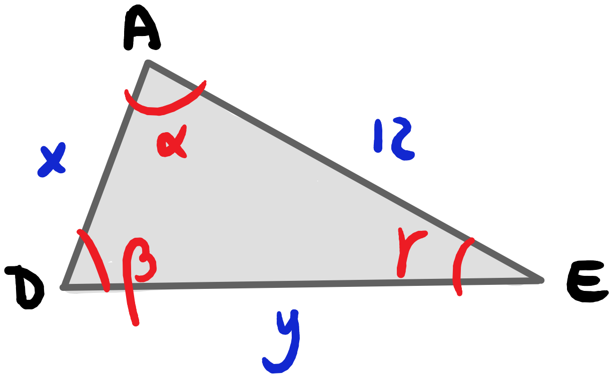
EXEMPLO

SABENDO QUE BC // DE, CALCULE OS VALORES DESCONHECIDOS.



$\Delta ABC \sim \Delta DEF$





$$K = \frac{18}{12} \rightarrow \underline{K = \frac{3}{2}}$$

$$x + 4 = x \cdot \frac{3}{2} \rightarrow 2x + 8 = 3x$$

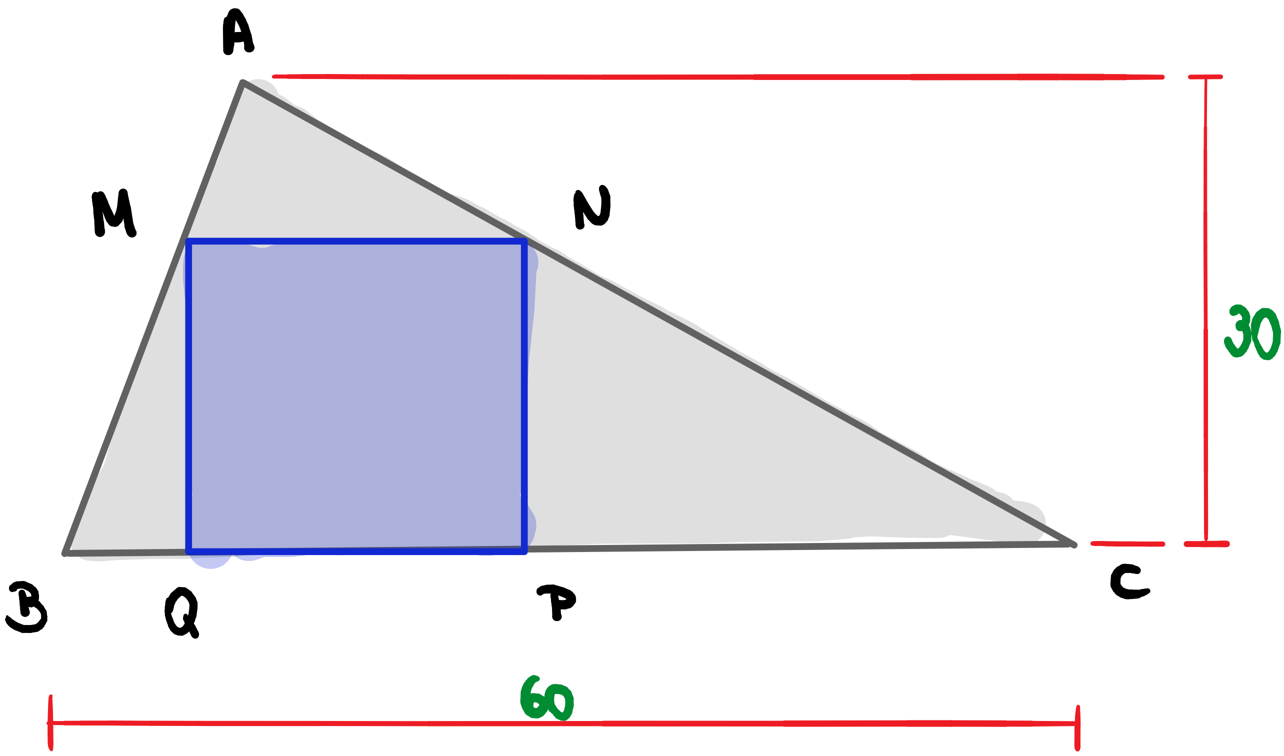
$$\underline{x = 8}$$

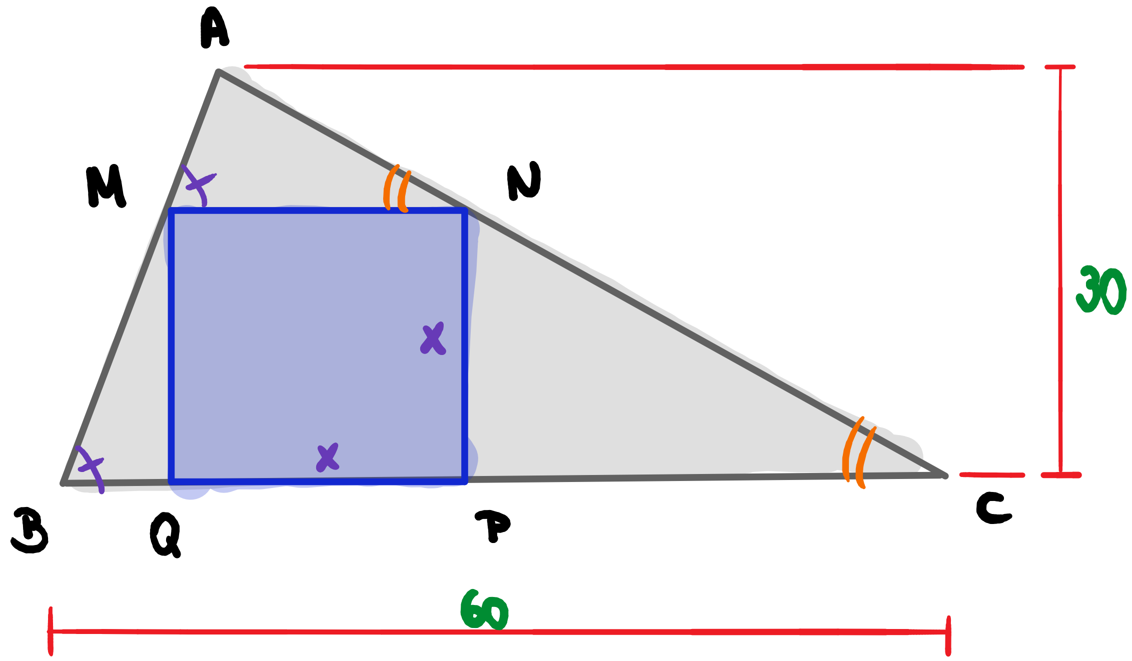
$$15 = y \cdot \frac{3}{2} \rightarrow y = \frac{\overset{5}{\cancel{15}} \cdot 2}{\cancel{3}}$$

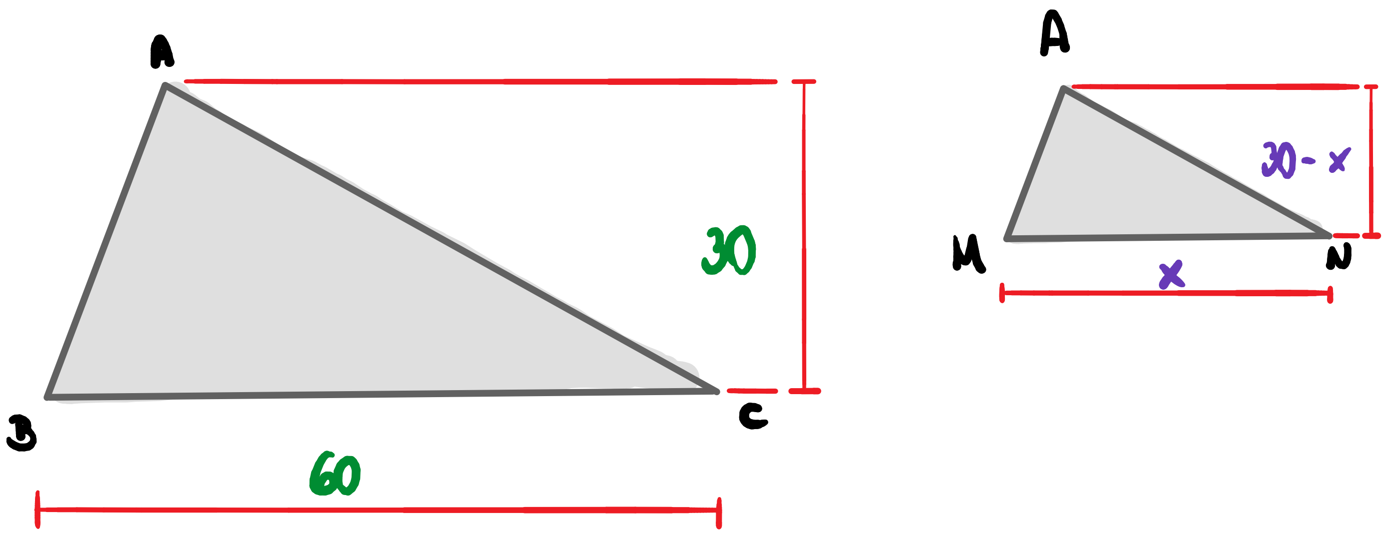
$$\underline{y = 10}$$

EXEMPLO

CALCULE O LADO DO QUADRADO MNPQ.







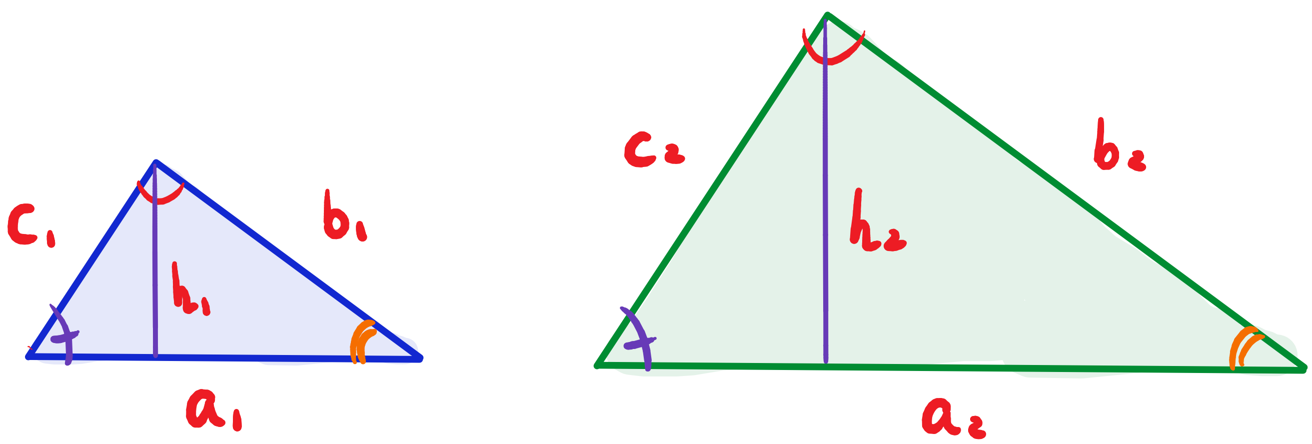
$$\frac{x}{\cancel{60}_{\,2}} = \frac{30 - x}{\cancel{30}_{\,1}} \rightarrow x = 60 - 2x$$

$$3x = 60$$

$$\underline{x = 20}$$

MEDIDAS CORRESPONDENTES

TODAS AS MEDIDAS LINEARES CORRESPONDENTES RESPEITAM A RAZÃO DE SEMELHANÇA, NÃO APENAS OS LADOS!!!



$$K = \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{h_1}{h_2} = \frac{p_1}{p_2} = \frac{R_1}{R_2}$$

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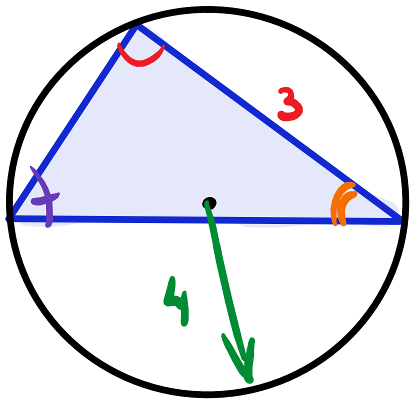
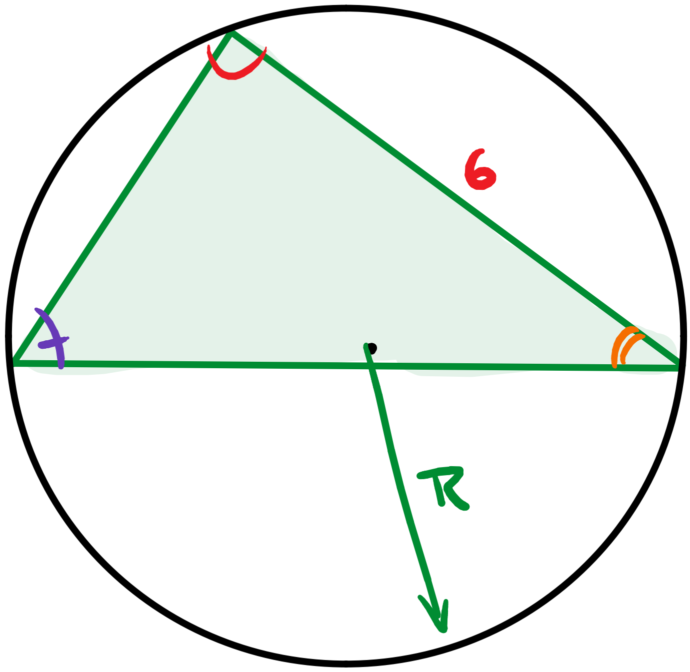
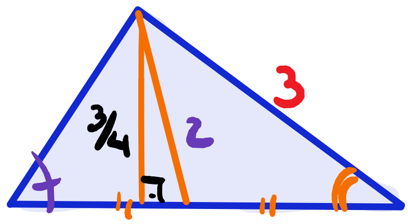
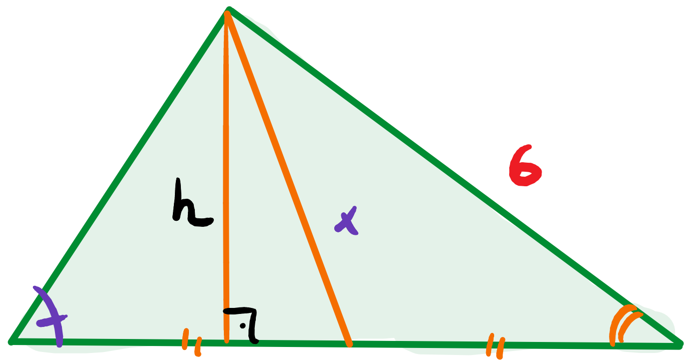




$K = 2$

$\frac{R}{4} = K \rightarrow \underline{R = 8}$





$K = \frac{6}{3} = 2$

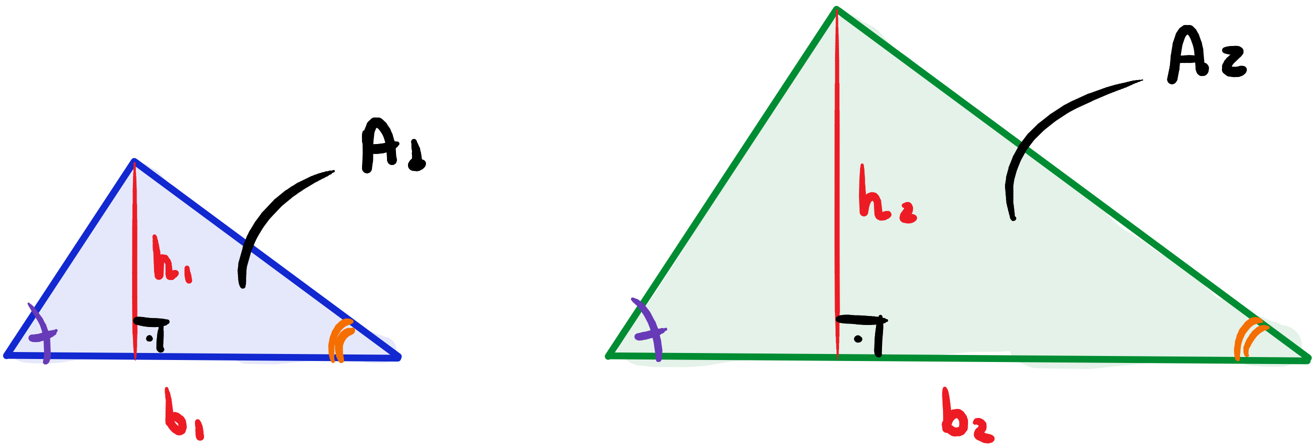
$x = 2.2$

$x = 4$

$h = 2 \cdot \frac{3}{4}$

$h = \frac{3}{2}$

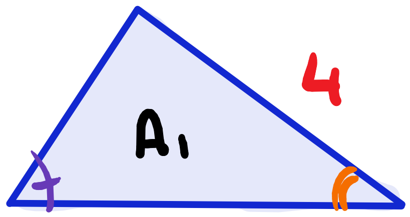
RAZÃO ENTRE ÁREAS

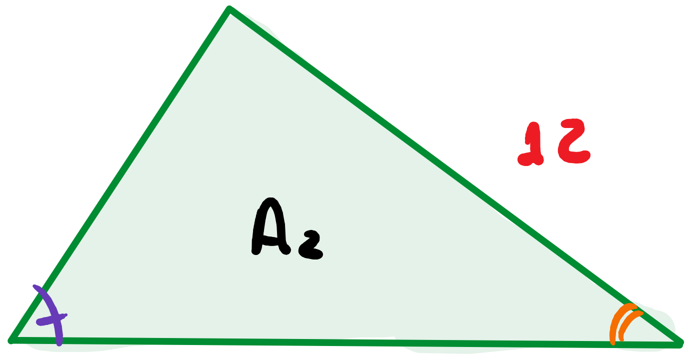


$$\frac{A_1}{A_2} = \frac{\frac{b_1 . h_1}{\cancel{2}}}{\frac{b_2 . h_2}{\cancel{2}}} = \frac{b_1 . h_1}{b_2 . h_2}$$

$$\frac{A_1}{A_2} = \overset{K}{\boxed{\frac{b_1}{b_2}}} . \overset{K}{\boxed{\frac{h_1}{h_2}}} = K . K$$

$$\boxed{\frac{A_1}{A_2} = K^2}$$

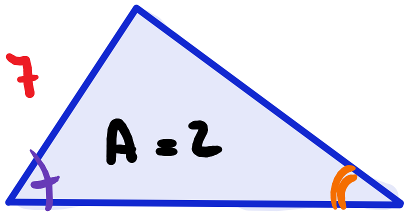


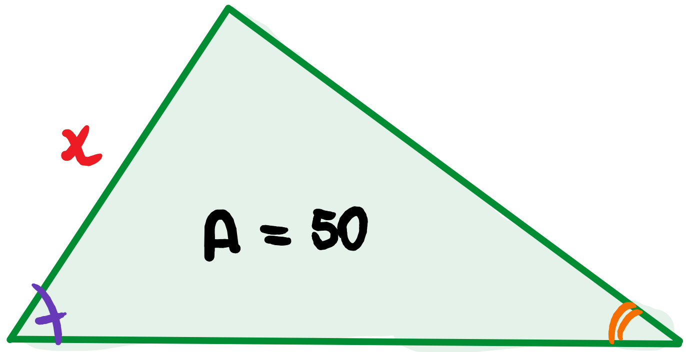


$$K = \frac{12}{4} = 3$$

$$\frac{A_2}{A_1} = 3^2$$

$$\frac{A_2}{A_1} = 9$$





$$\frac{A_2}{A_1} = K^2 \rightarrow K^2 = \frac{50}{2} \rightarrow K^2 = 25$$

$$\underline{K = 5}$$

$$x = 5 \cdot 7 \rightarrow \underline{x = 35}$$

EXEMPLO

DOIS TRIÂNGULOS SEMELHANTES POSSUEM BASES CORRESPONDENTES MEDINDO 2 E 6, RESPECTIVAMENTE.

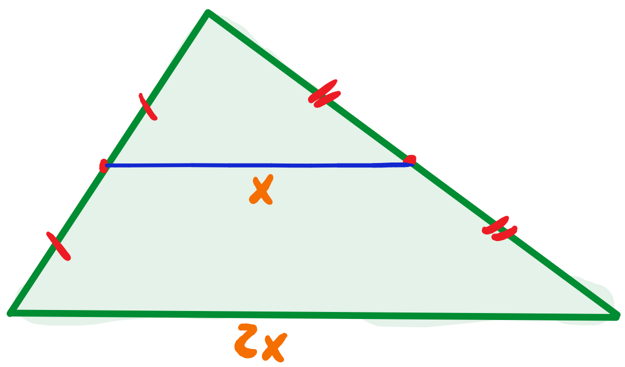
SE A ÁREA DO MENOR TRIÂNGULO É 7, QUAL A ÁREA DO MAIOR?

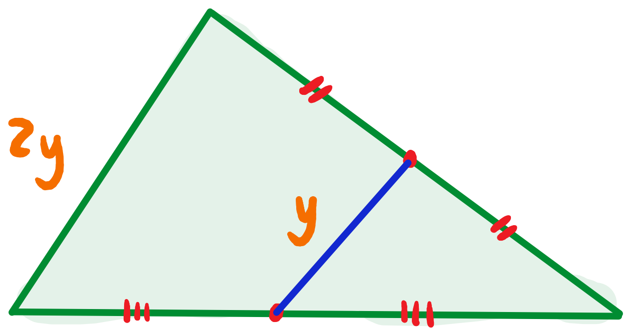
$$K = \frac{b_2}{b_1} \rightarrow K = \frac{6}{2} \rightarrow \underline{K = 3}$$

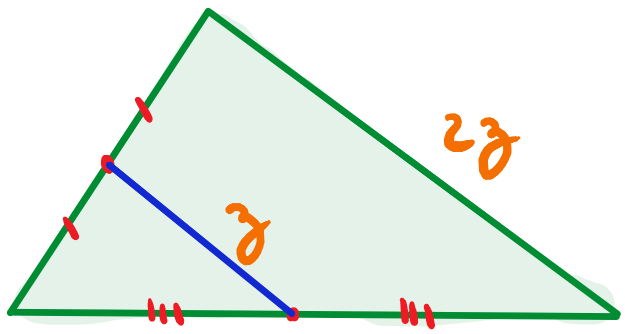
$$\frac{A_2}{A_1} = K^2 \rightarrow \frac{A_2}{7} = 3^2 \rightarrow \underline{A_2 = 63}$$

BASE MÉDIA

SEGMENTO LIGANDO
PONTOS MÉDIOS
DOS LADOS DE UM TRIÂNGULO







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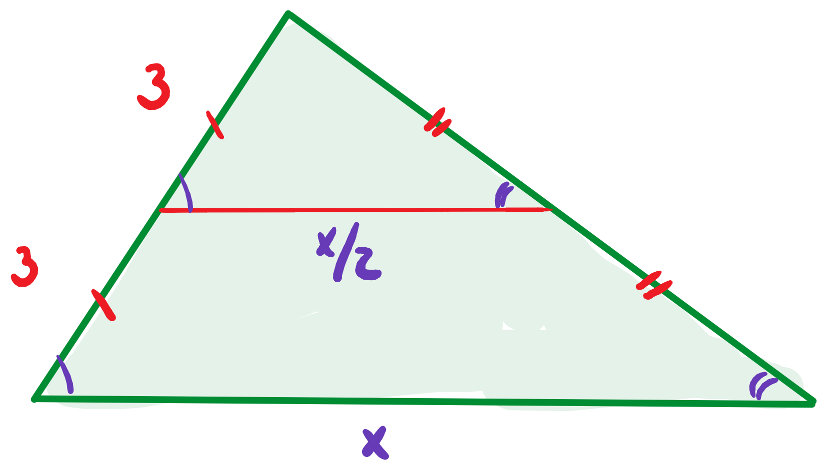
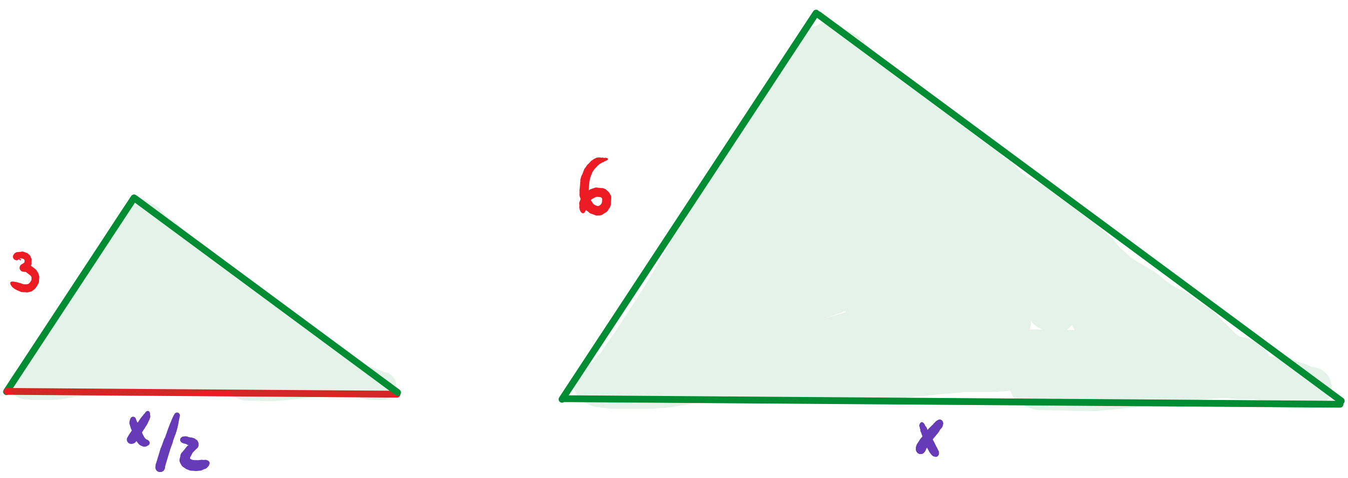
RAZÃO DE SEMELHANÇA – BASE MÉDIA



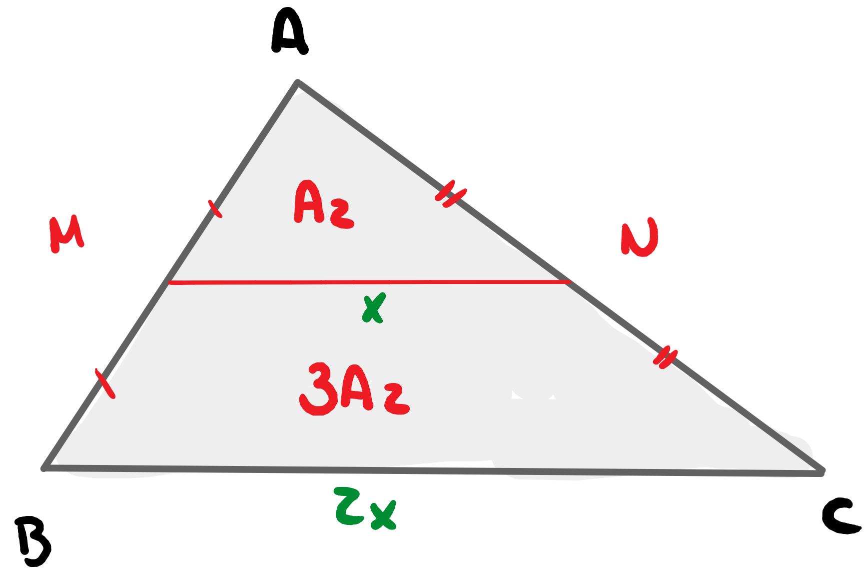


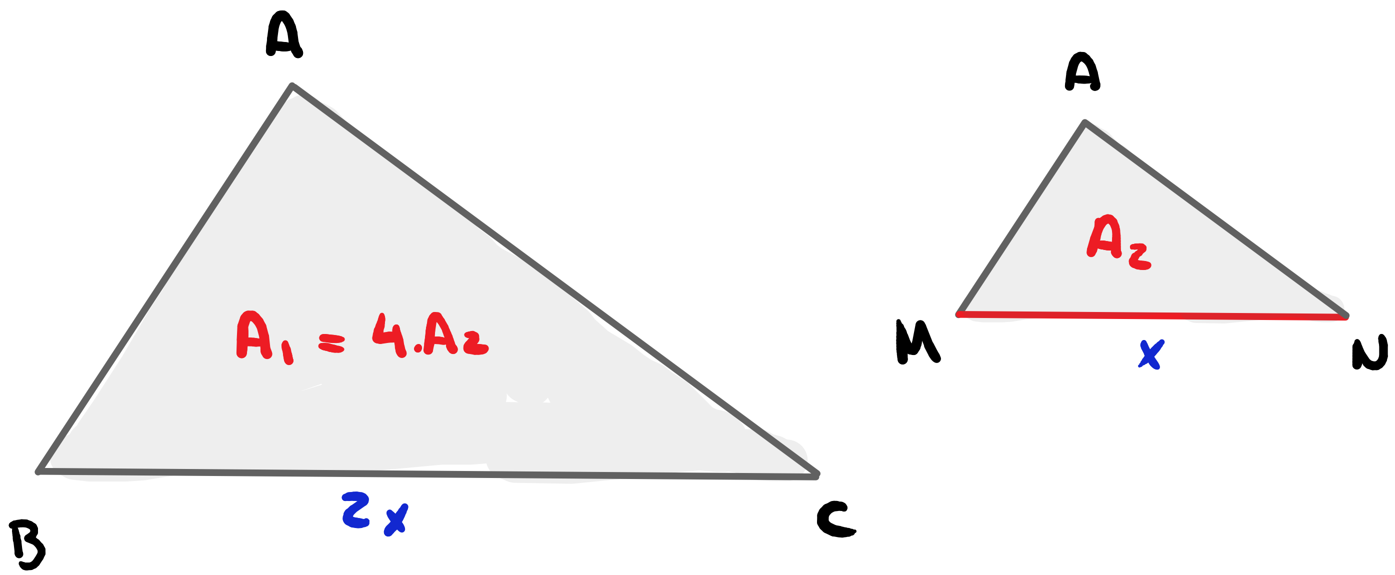
$$k = \frac{1}{2}$$

EXEMPLO

SEJA O TRIÂNGULO ABC. M E N SÃO PONTOS MÉDIOS, DOS LADOS AB E AC, RESPECTIVAMENTE.

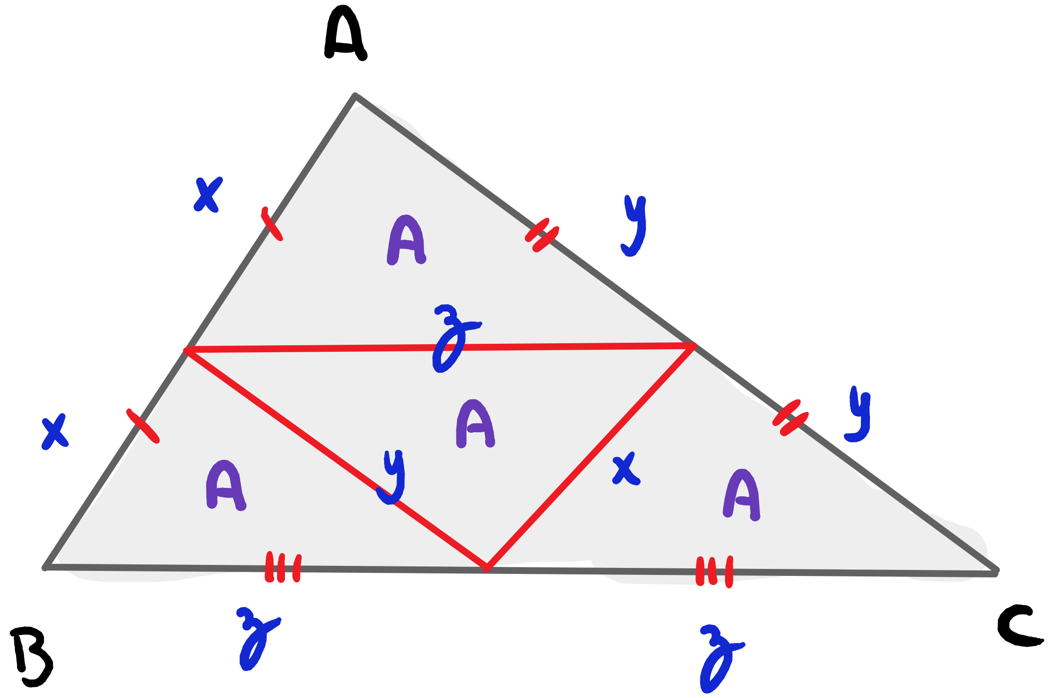
CALCULE A RAZÃO ENTRE A ÁREA DO TRIÂNGULO ABC E DO QUADRILÁTERO BCNM.





$$K = 2 \rightarrow \frac{A_1}{A_2} = 2^2 \rightarrow \frac{A_1}{A_2} = 4$$

$$\frac{A_{ABC}}{A_{BCNM}} = \frac{4\,\cancel{A_2}}{3\,\cancel{A_2}} \rightarrow \boxed{\frac{A_{ABC}}{A_{BCNM}} = \frac{4}{3}}$$



$$\frac{4A}{3A} = \frac{4}{3}$$

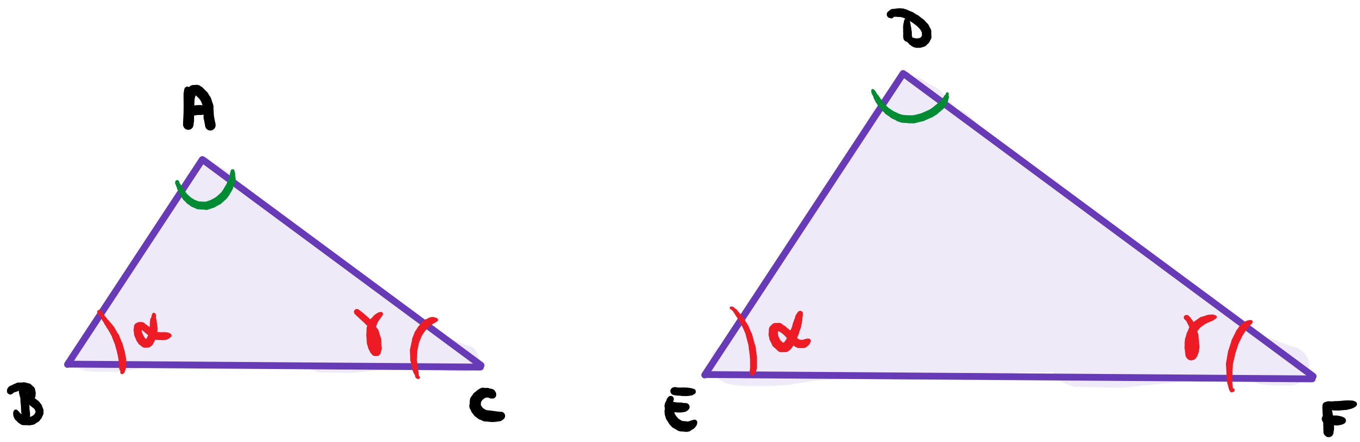
CASOS DE SEMELHANÇA

SÃO CONDIÇÕES SUFICIENTES PARA SE GARANTIR A SEMELHANÇA DE TRIÂNGULOS.

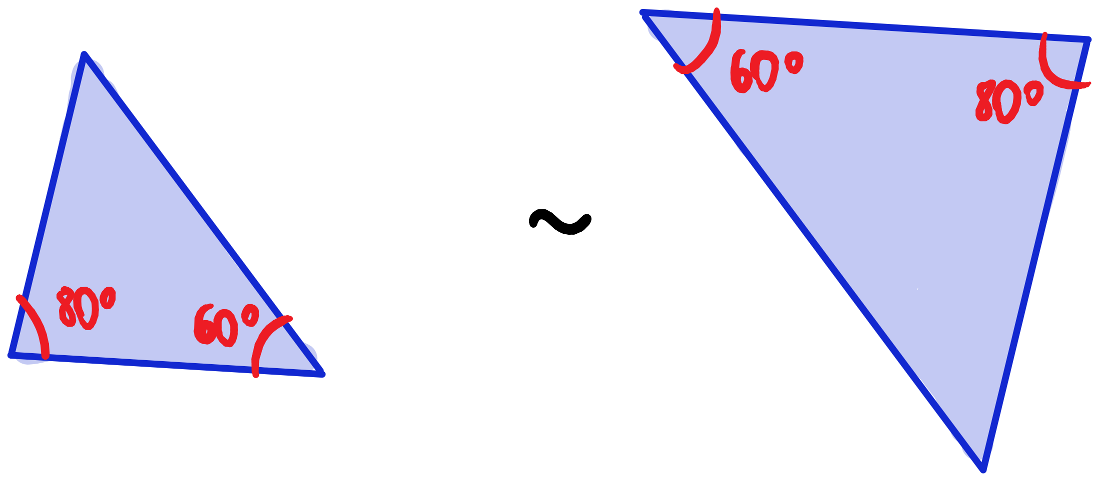
CASOS PRINCIPAIS
- AA
- LLL
- LAL

CASO AA

2 ÂNGULOS IGUAIS ⇒ 3 ÂNGULOS IGUAIS

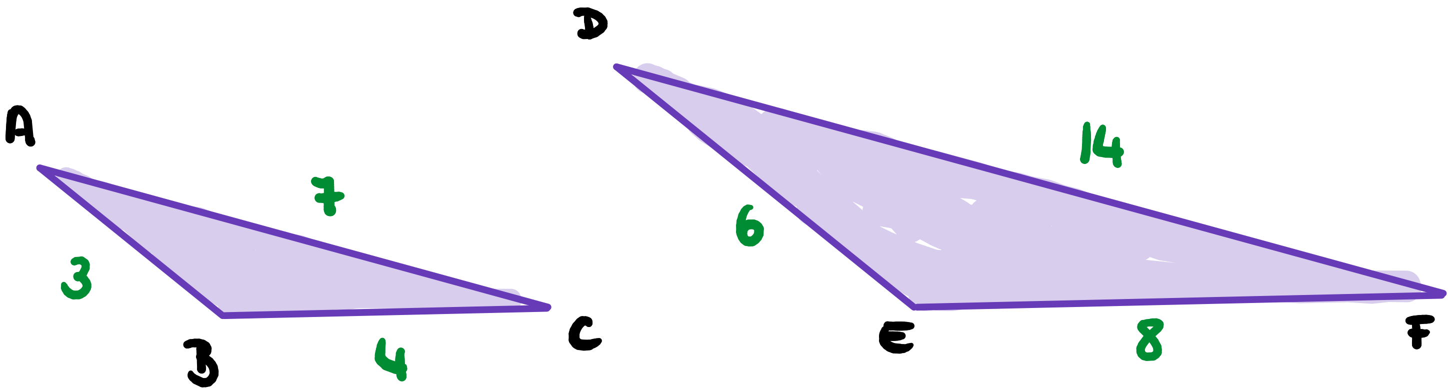


$\boxed{\begin{array}{l} \hat{B} = \hat{E} \\ \hat{C} = \hat{F} \end{array}} \longrightarrow \triangle ABC \sim \triangle DEF$

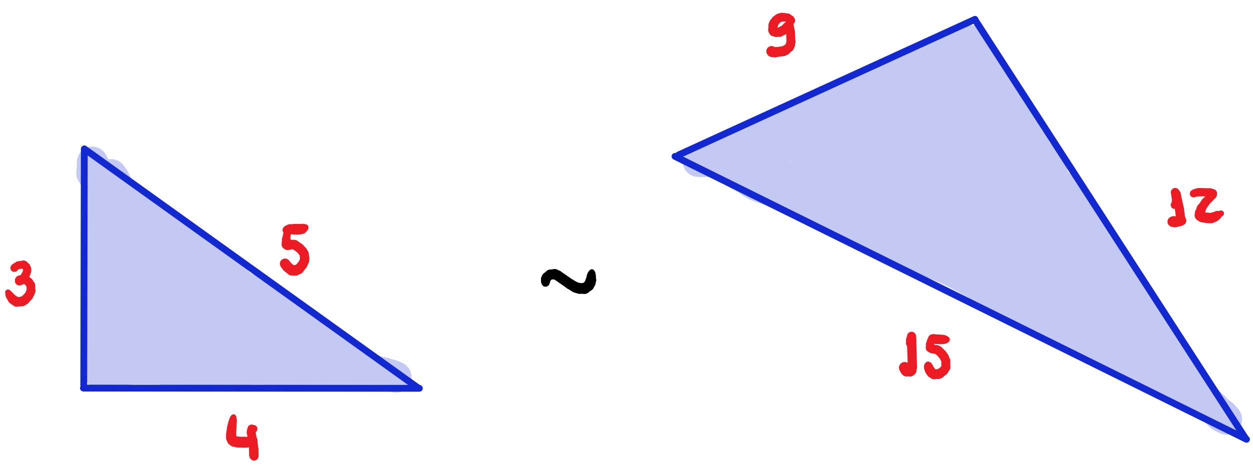


CASO LLL

3 LADOS PROPORCIONAIS



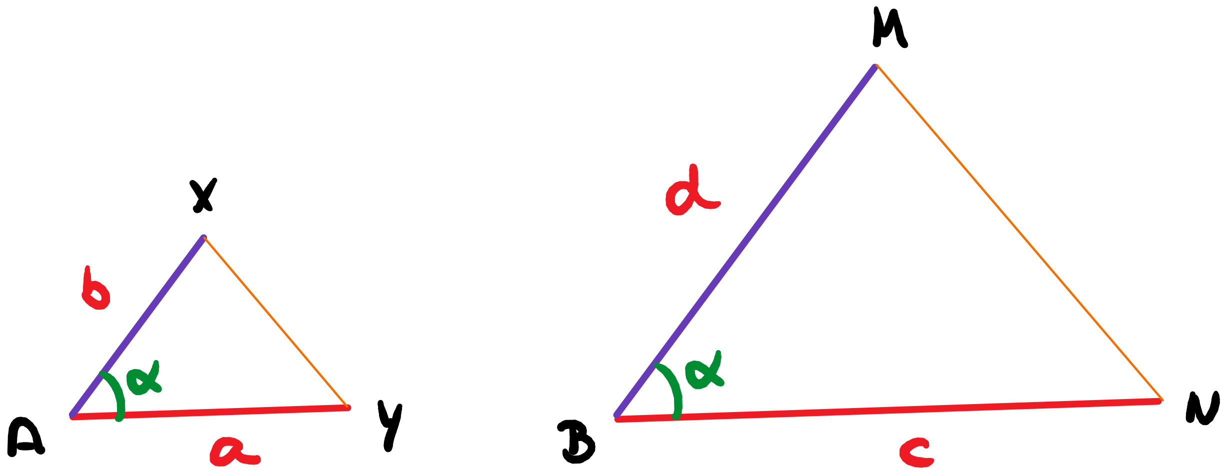
$$\frac{14}{7} = \frac{8}{4} = \frac{6}{3} = 2 \rightarrow \Delta ABC \sim \Delta DEF$$



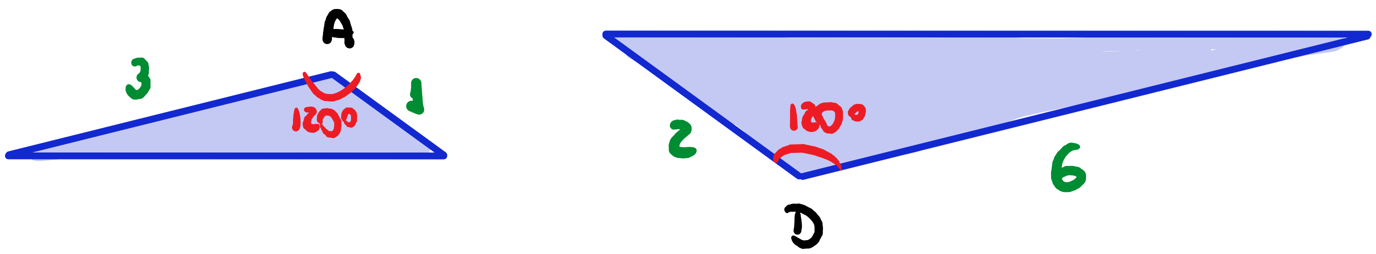
$$\frac{15}{5} = \frac{12}{4} = \frac{9}{3} = 3$$

CASO LAL

ÂNGULO IGUAL ENTRE LADOS PROPORCIONAIS



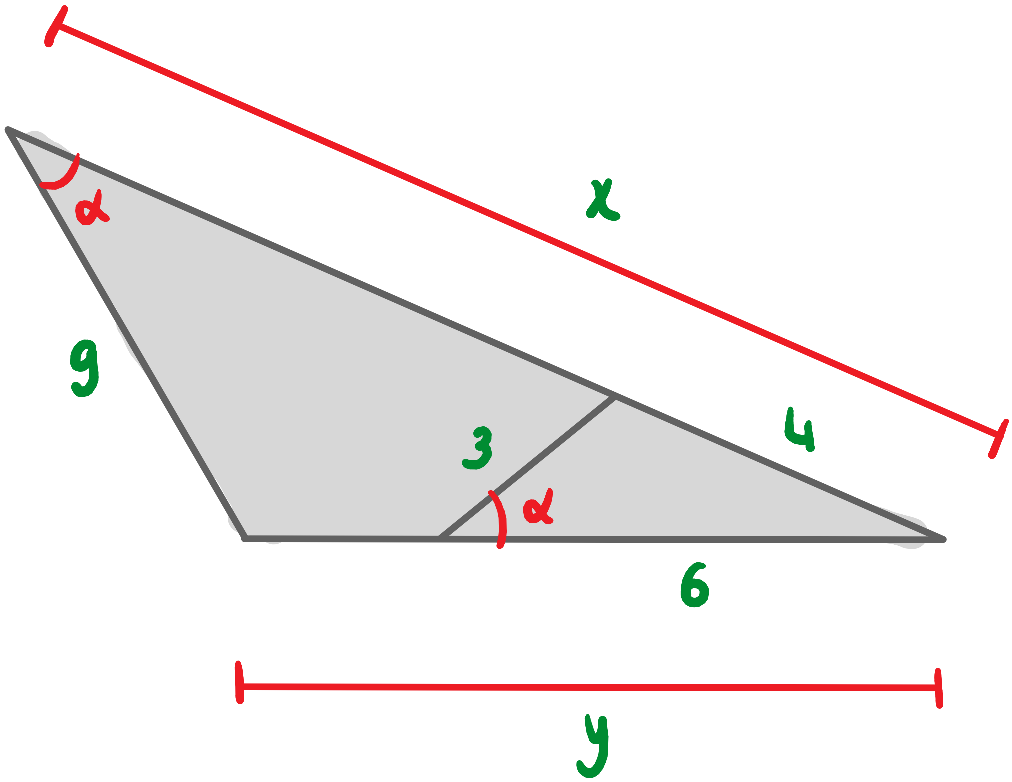
$\frac{a}{c} = \frac{b}{d}$; $\tilde{A} = \tilde{B}$ $\longrightarrow$ $\Delta AXY \sim \Delta BMN$

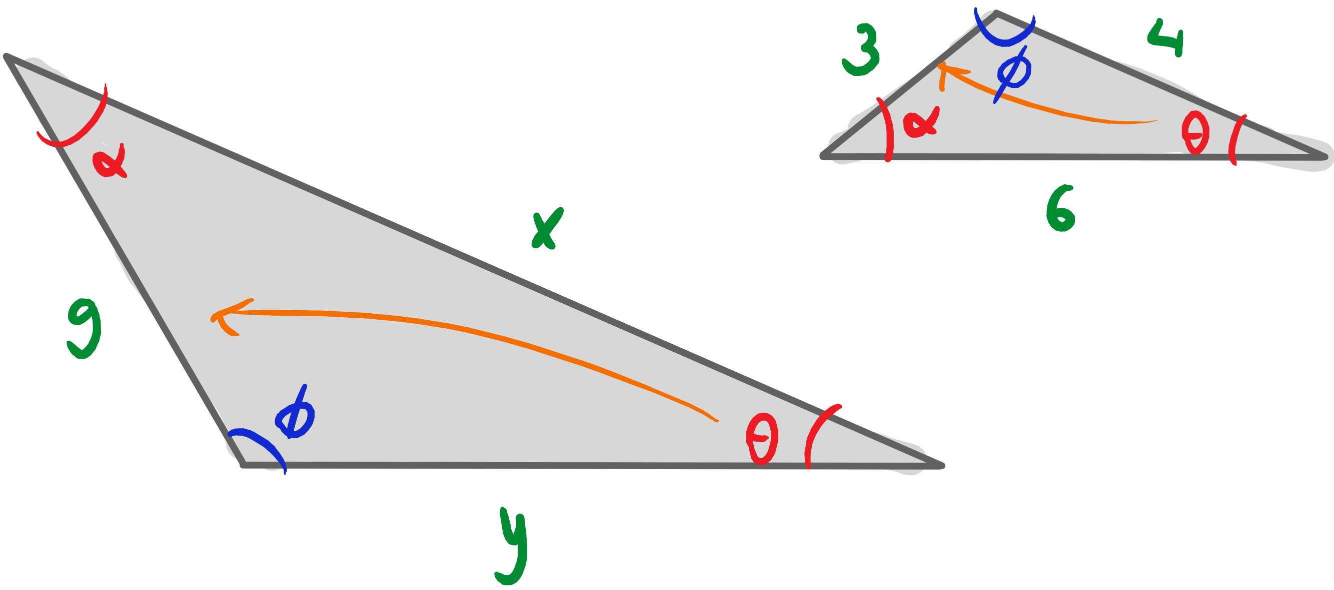


$\frac{6}{3} = \frac{2}{1}$; $120° = 120°$ $\longrightarrow$ Δ SEMELHANTES

EXEMPLO

DETERMINE O VALOR DE x E y NA FIGURA.





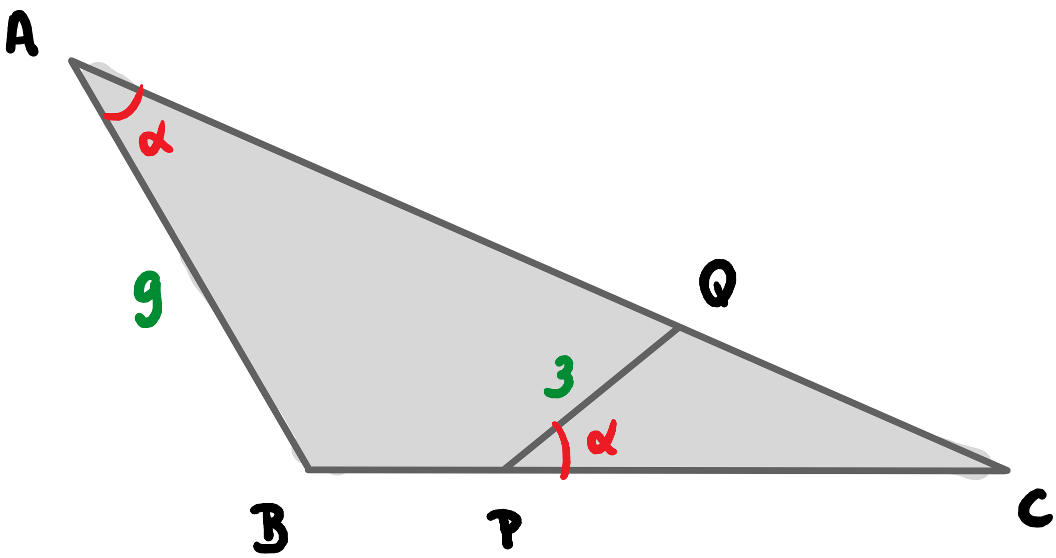
$$K = \frac{9}{3} \rightarrow \underline{K = 3}$$

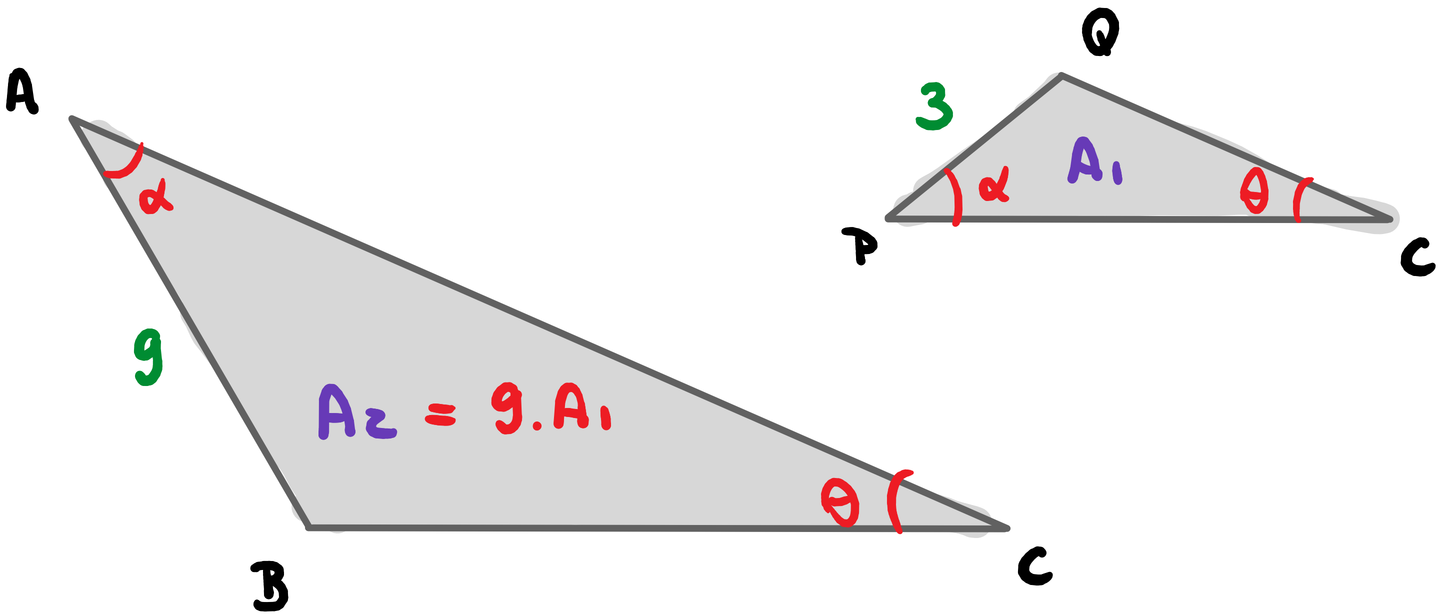
$$x = 6.3 \rightarrow \underline{x = 18}$$

$$y = 4.3 \rightarrow \underline{y = 12}$$

EXEMPLO

CALCULE A RAZÃO ENTRE A ÁREA DO QUADRILÁTERO ABPQ E A ÁREA DO TRIÂNGULO ABC.

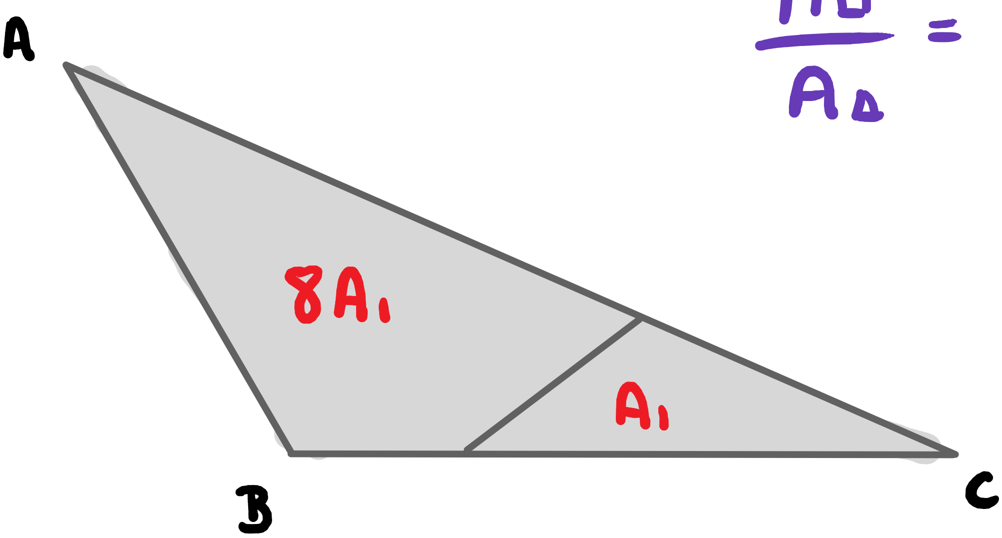




$$K = \frac{9}{3} \rightarrow \underline{K = 3}$$

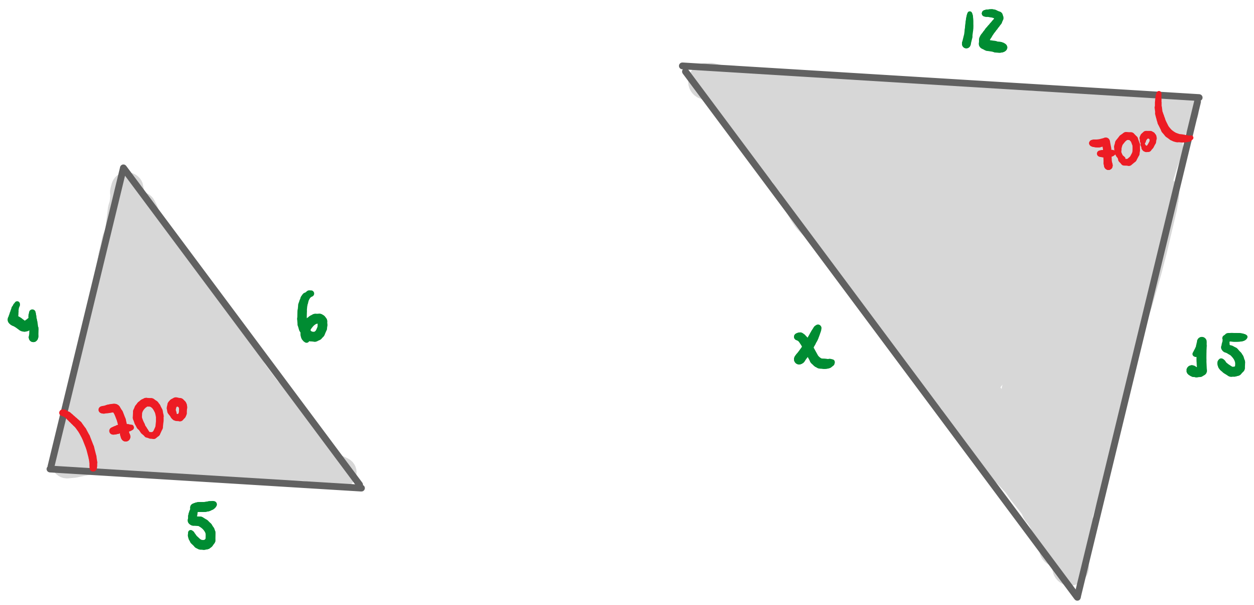
$$\frac{A_2}{A_1} = K^2 \rightarrow \frac{A_2}{A_1} = 3^2 \rightarrow \underline{A_2 = 9.A_1}$$

$$\frac{A_{\square}}{A_{\triangle}} = \frac{8A_1}{9A_1} = \underline{\frac{8}{9}}$$



EXEMPLO

CALCULE A MEDIDA DO LADO x.



$$\frac{15}{5} = 3 \quad ; \quad \frac{12}{4} = 3 \quad K$$

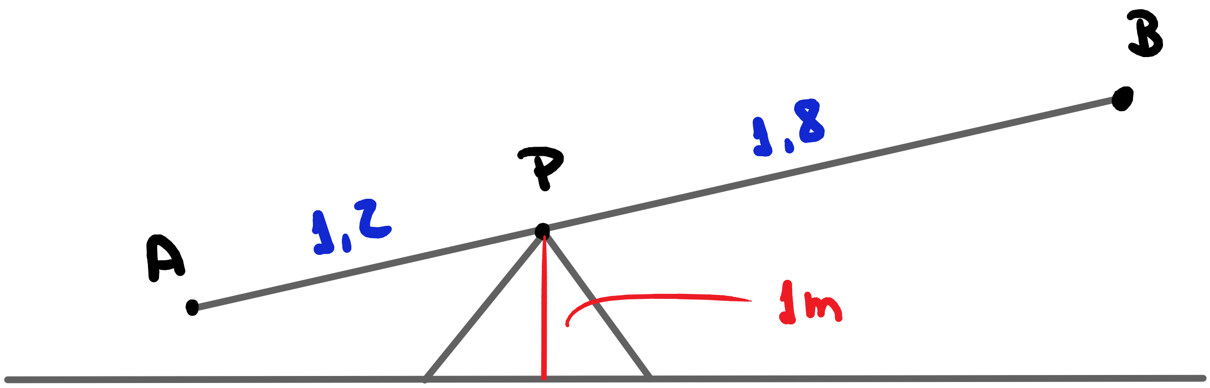
LAL

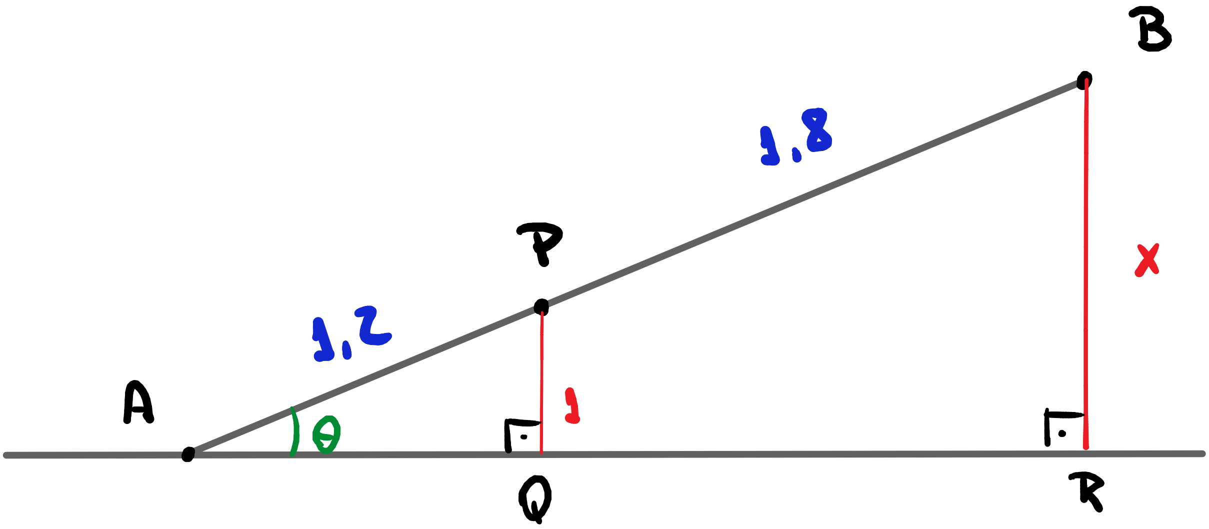
$$x = 6.3 \rightarrow \underline{x = 18}$$

EXEMPLO

A FIGURA APRESENTA UMA GANGORRA QUE GIRA EM TORNO DO PONTO P. A ALTURA DO SUPORTE É 1m.

CALCULE A ALTURA QUE UMA PESSOA SENTADA NO PONTO B ATINGE QUANDO A PESSOA LOCALIZADA NO PONTO Â ESTÁ NO CHÃO.





$$\Delta ABR \sim \Delta APQ$$

$$\frac{x}{1} = \frac{3,0}{1,2}$$

$$\frac{x}{1} = \frac{\cancel{30}^{\,5}}{\cancel{12}_{\,2}}$$

$$x = \frac{5}{2}$$

$$\underline{x = 2,5\,m}$$

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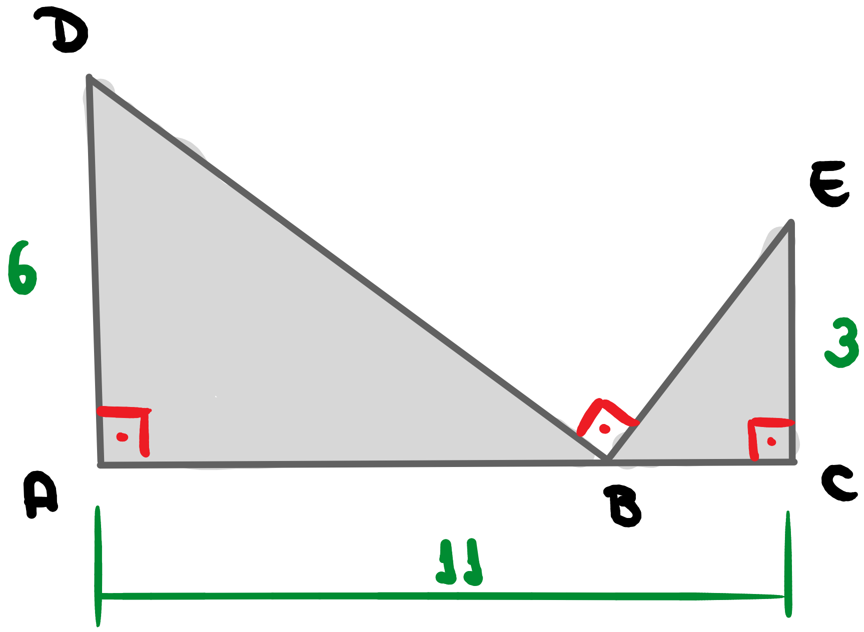
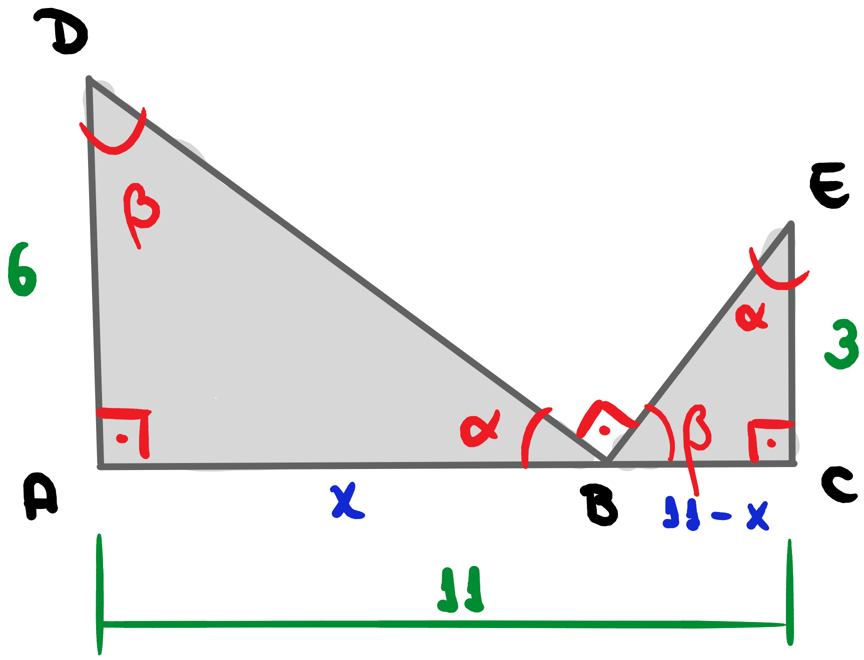
EXEMPLO

CALCULE OS POSSÍVEIS VALORES DE AB NA FIGURA ABAIXO.





$$\frac{x}{3} = \frac{6}{11 - x}$$

$$11x - x^2 = 18$$

$$x^2 - 11x + 18 = 0$$

$\boxed{x = 2}$ ou $\boxed{x = 9}$

$$\Delta = (-11)^2 - 4(1)(18) \longrightarrow \Delta = 49$$

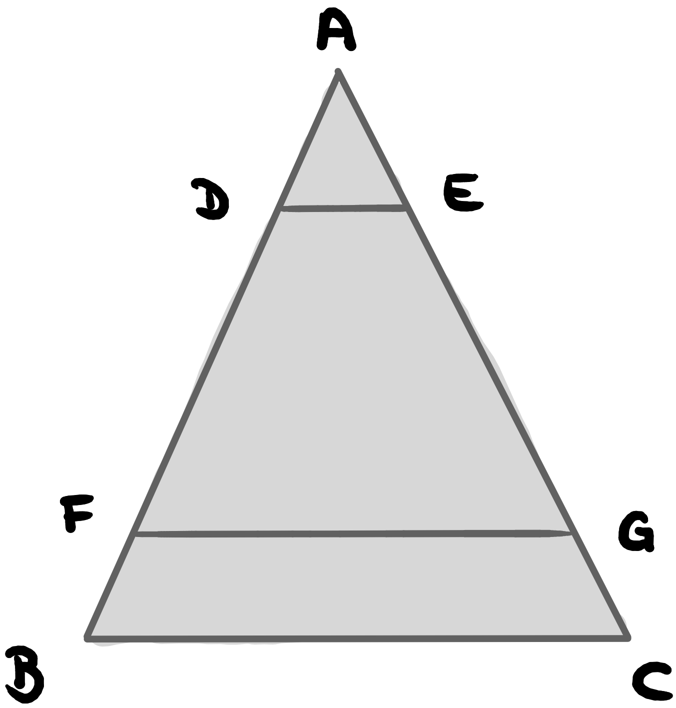
$$x = \frac{-(-11) \pm \sqrt{49}}{2 \cdot 1} \longrightarrow x = 2 \; ; \; x = 9$$

EXEMPLO

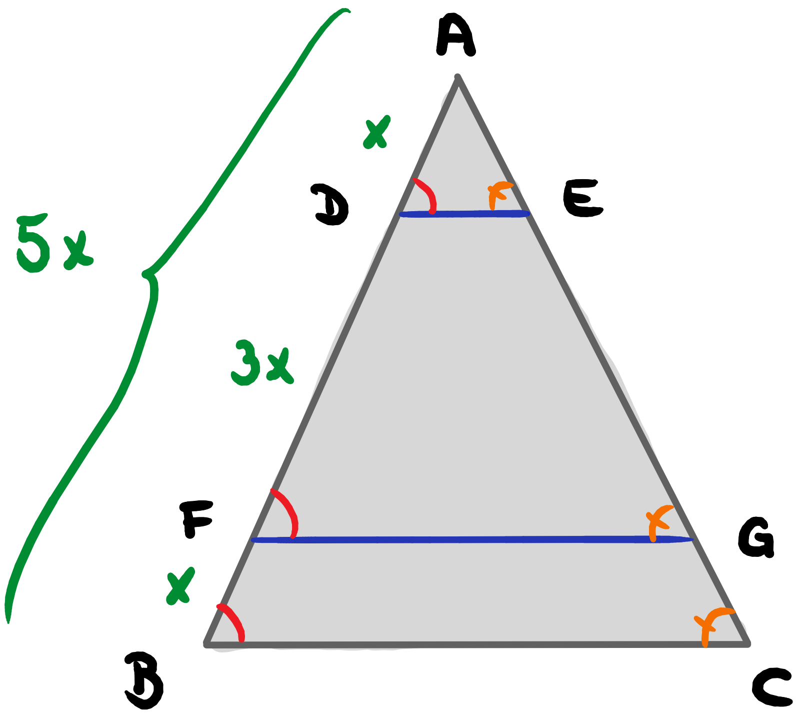
NA FIGURA ABAIXO, DE // FG // BC.

ALÉM DISSO, AB = 5.AD = 5.BF

CALCULE O VALOR DA RAZÃO FG/DE.



$$\underbrace{AB}_{5x} = 5\,\underbrace{AD}_{x} = 5\,\underbrace{BF}_{x}$$



$$\Delta AFG \sim \Delta ADE$$

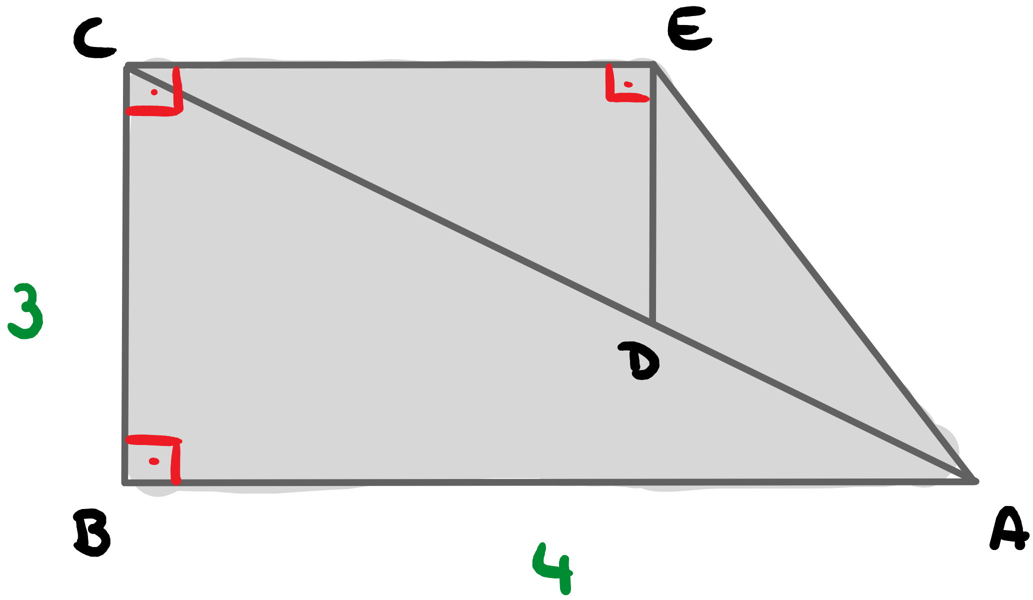
$$\frac{FG}{DE} = \frac{4\cancel{x}}{\cancel{x}}$$

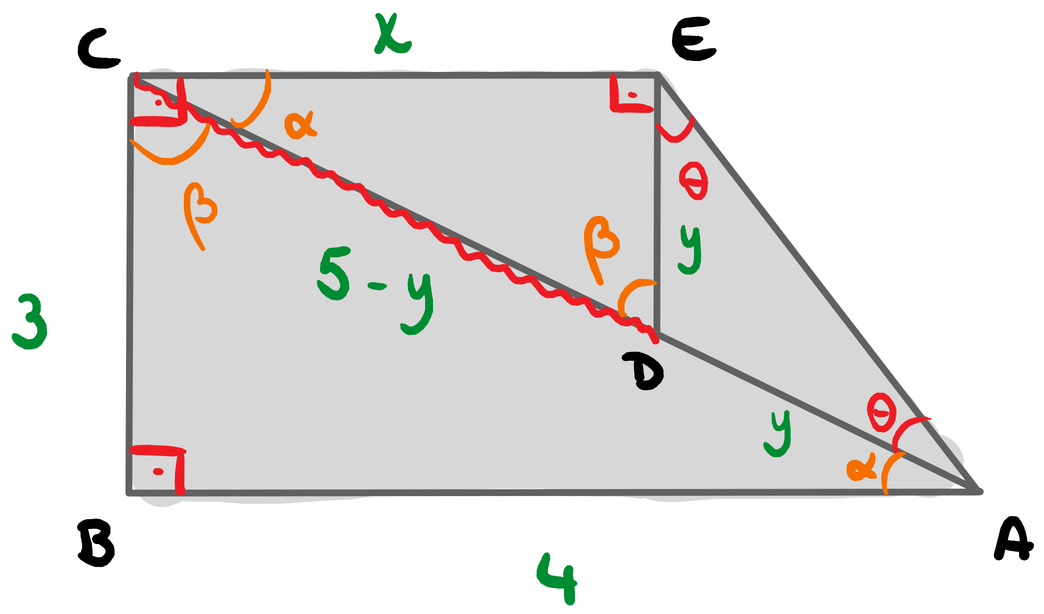
$$\boxed{\frac{FG}{DE} = 4}$$

EXEMPLO

NA FIGURA, EÂD = AÊD.

CALCULE O COMPRIMENTO CE.





$$\Delta ABC \rightarrow AB^2 = 3^2 + 4^2 \rightarrow \underline{AB = 5}$$

$$\underline{\Delta ABC \sim \Delta CDE}$$

$$\frac{y}{3} = \frac{5-y}{5}$$

$$5y = 15 - 3y$$

$$8y = 15$$

$$\underline{y = \frac{15}{8}}$$

$$\Delta ABC \sim \Delta CDE$$

$$\frac{x}{4} = \frac{15/8}{3}$$

$$3x = \cancel{4} \cdot \frac{15}{\cancel{8}\,2}$$

$$x = \frac{\cancel{15}\ 5}{\cancel{3} \cdot 2}$$

$$\boxed{x = \frac{5}{2}}$$

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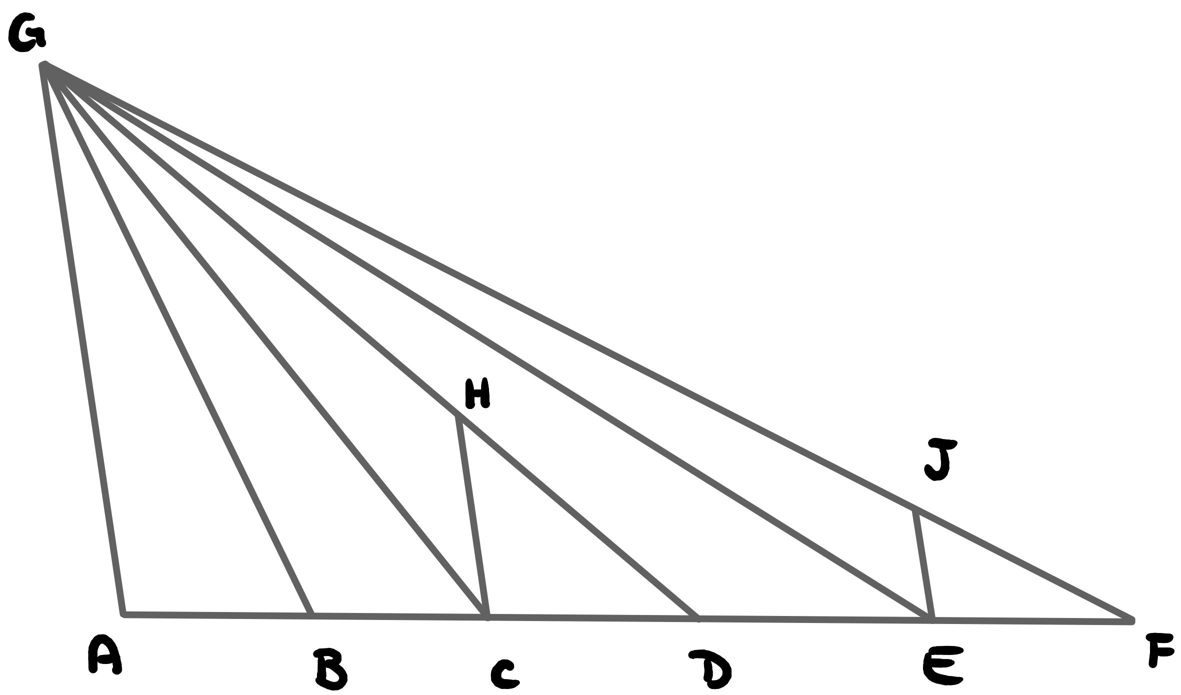
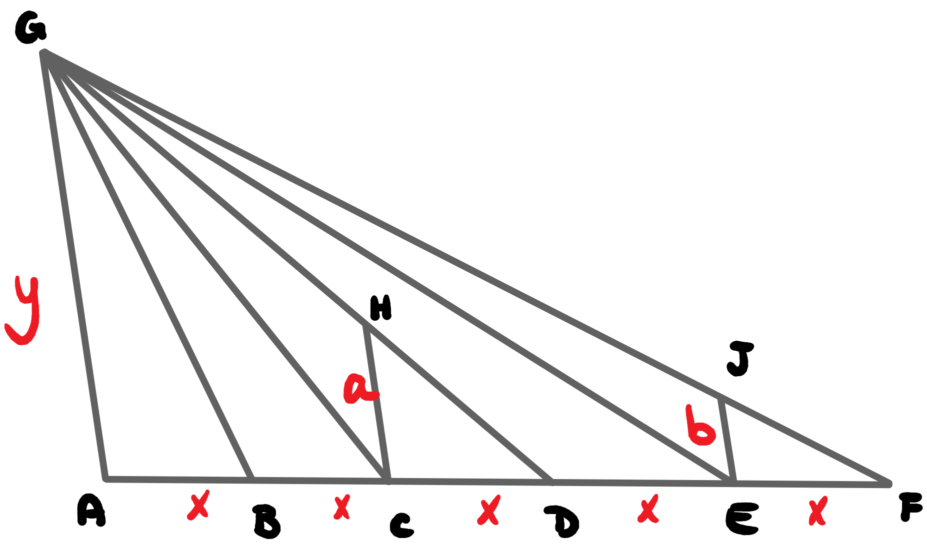
EXEMPLO

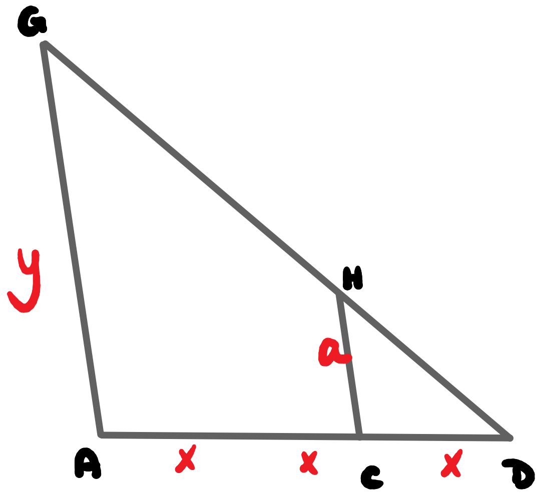
NA FIGURA, O LADO AF FOI DIVIDIDO EM PARTES IGUAIS PELOS PONTOS B, C, D e E.

SABENDO QUE AG // CH // JE, CALCULE CH/EJ.



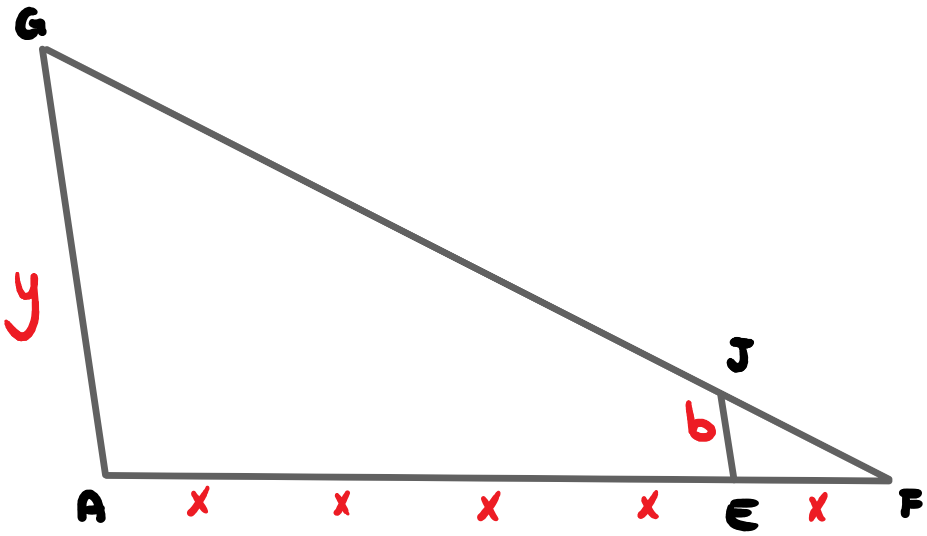
$$\frac{a}{b} = ?$$





$$\frac{a}{y} = \frac{\cancel{x}}{3\cancel{x}}$$

$$\boxed{a = \frac{y}{3}}$$



$$\frac{b}{y} = \frac{\cancel{x}}{5\cancel{x}}$$

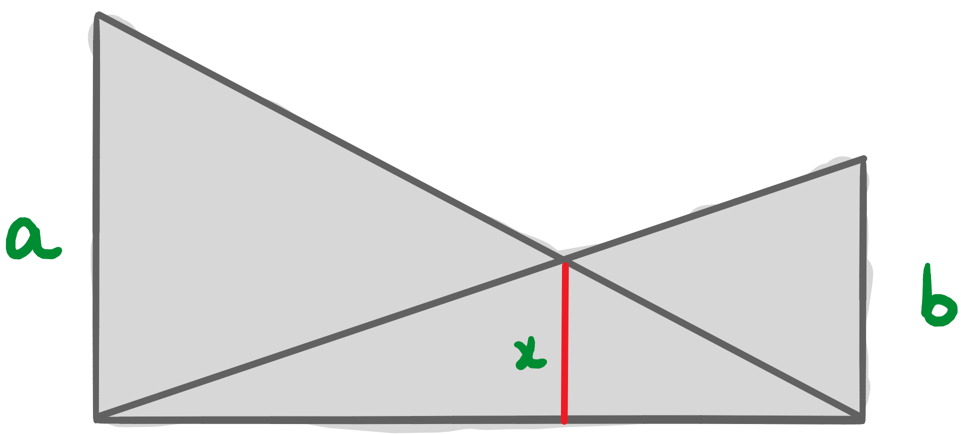
$$\boxed{b = \frac{y}{5}}$$

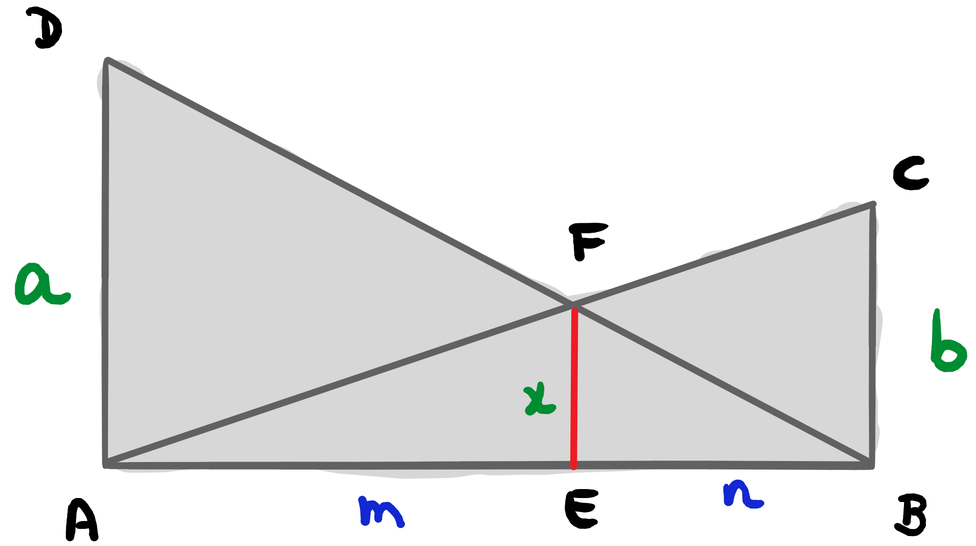
$$\frac{a}{b} = \frac{y/3}{y/5} = \frac{\cancel{y}}{3} \cdot \frac{5}{\cancel{y}} \rightarrow \boxed{\frac{a}{b} = \frac{5}{3}}$$

EXEMPLO

NA FIGURA, MOSTRE QUE $1/x = 1/a + 1/b$.

$$(a \parallel b \parallel x)$$





$\Delta FEB \sim \Delta DAB$ $\qquad$ $\Delta AFE \sim \Delta ACB$

$$\frac{x}{a} = \frac{n}{m+n} \qquad \frac{x}{b} = \frac{m}{m+n}$$

$$\frac{x}{a} + \frac{x}{b} = \frac{n}{m+n} + \frac{m}{m+n}$$

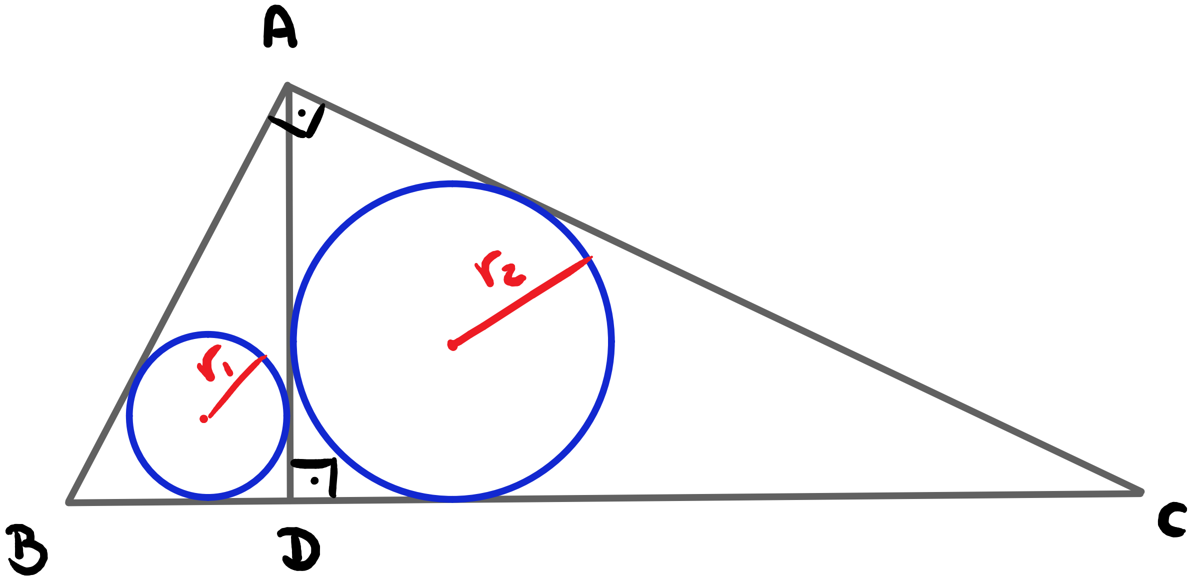
$$\frac{x}{a} + \frac{x}{b} = \frac{m+n}{m+n} = 1$$

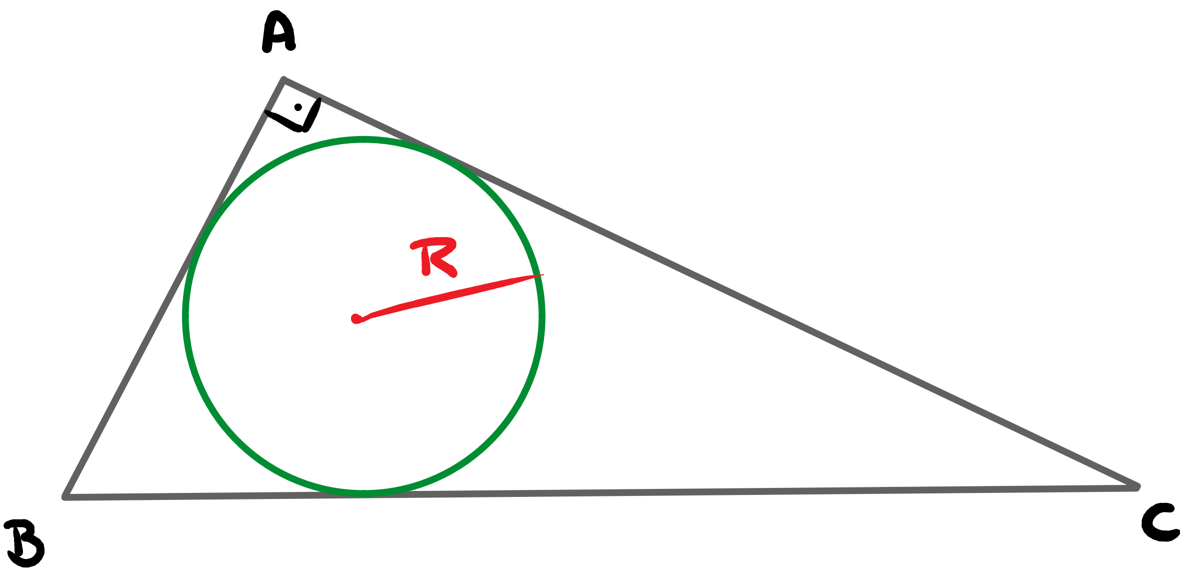
$$\frac{x}{a} + \frac{x}{b} = 1$$

$$x\left(\frac{1}{a} + \frac{1}{b}\right) = 1 \longrightarrow \boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{x}}$$

EXEMPLO

CONSIDERANDO A FIGURA, EXPRESSE O VALOR DE R EM FUNÇÃO DE r_1 E r_2.

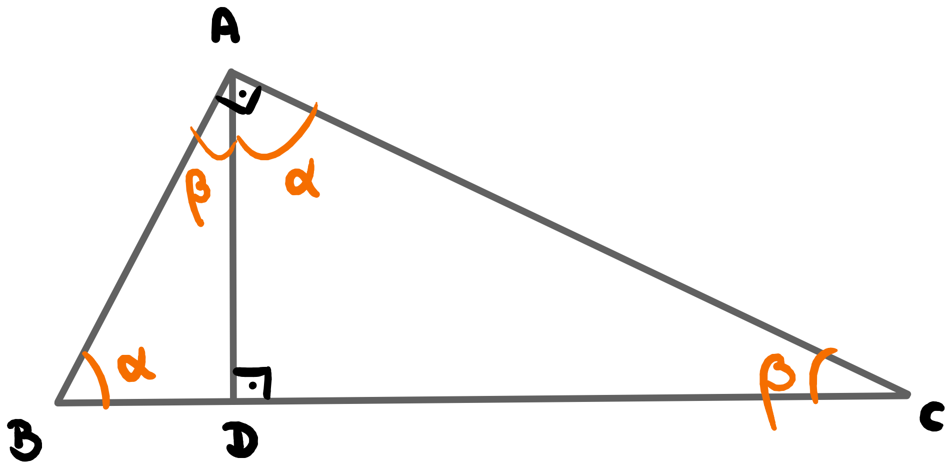




DICA: ÁREA → $A = p \cdot r$

r → RAIO INSCRITA

p → SEMI PER.



$$A_T = A_1 + A_2$$

$$\frac{\cancel{p_T} \cdot R}{\cancel{p_T}} = \frac{p_1 \cdot r_1}{p_T} + \frac{p_2 \cdot r_2}{p_T}$$

$$R = K_1 \cdot r_1 + K_2 \cdot r_2$$

$$R = \frac{r_1}{R} \cdot r_1 + \frac{r_2}{R} \cdot r_2$$

$$\boxed{R^2 = r_1^2 + r_2^2}$$

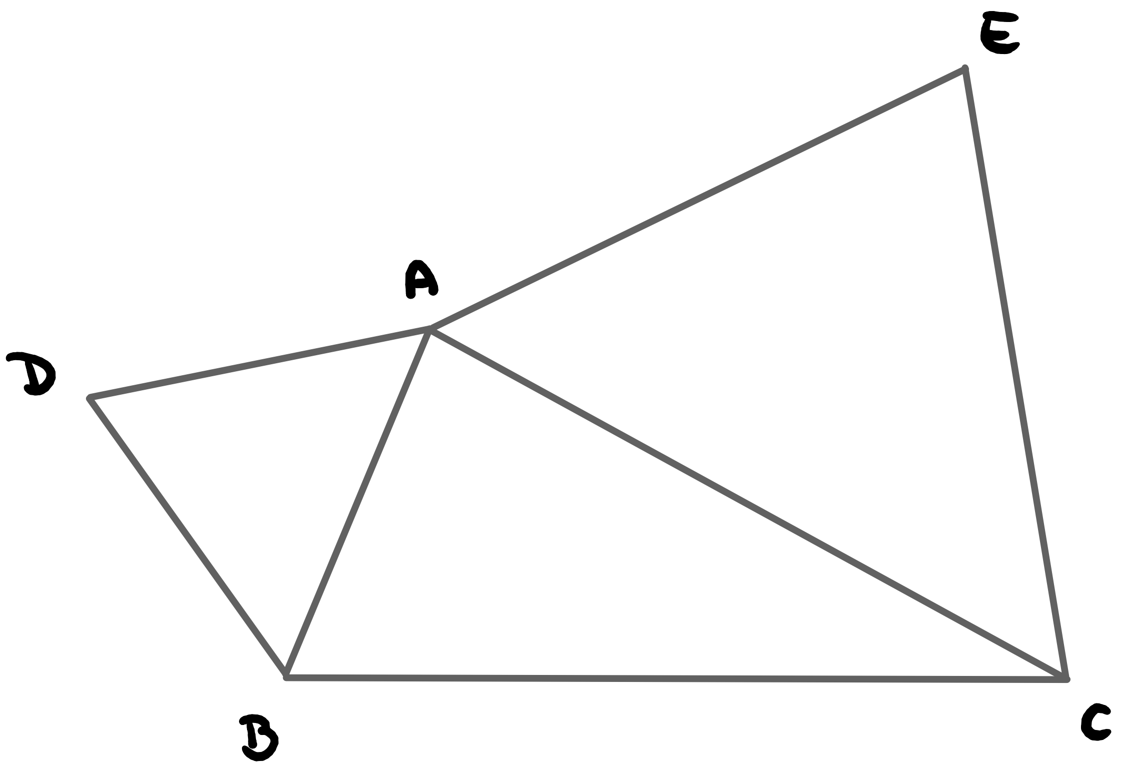
EXEMPLO

SEJA ABC UM TRIÂNGULO QUALQUER, COMO MOSTRA A FIGURA.

OS TRIÂNGULOS ABD E ACE SÃO EQUILÁTEROS.

OS PONTOS F E G SÃO PONTOS MÉDIOS DOS LADOS AD E AC, RESPECTIVAMENTE.

CALCULE O VALOR DA RAZÃO FG/ BE



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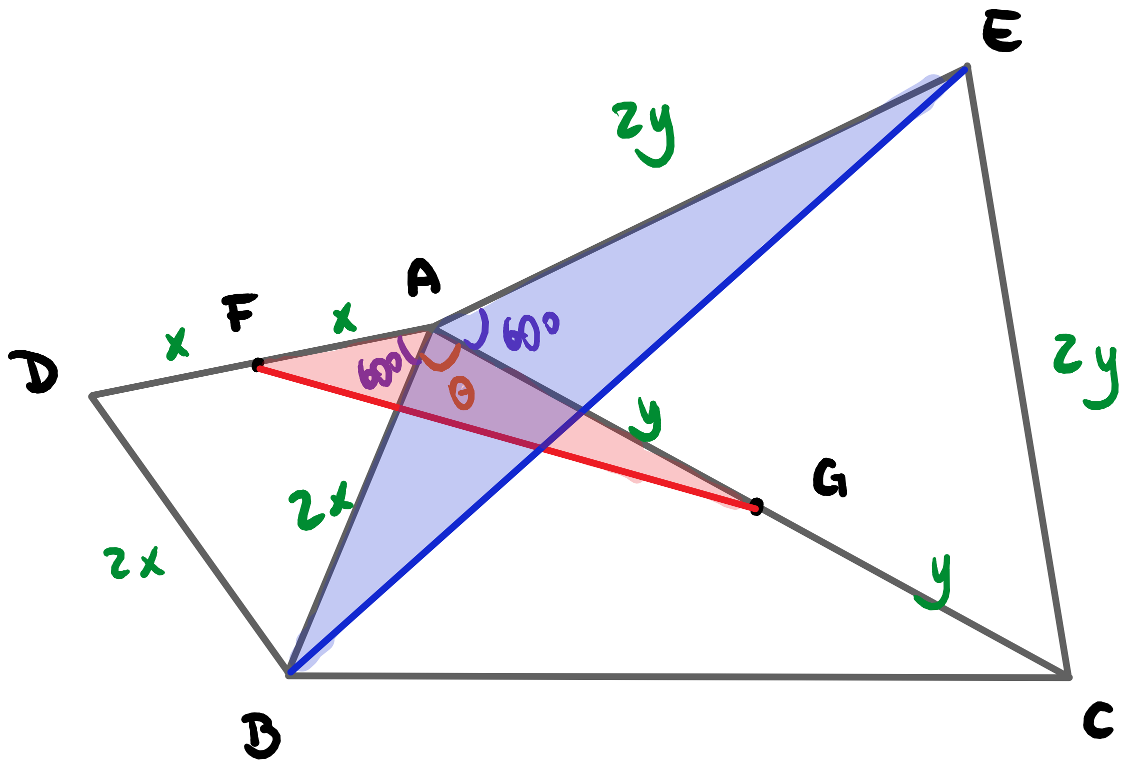

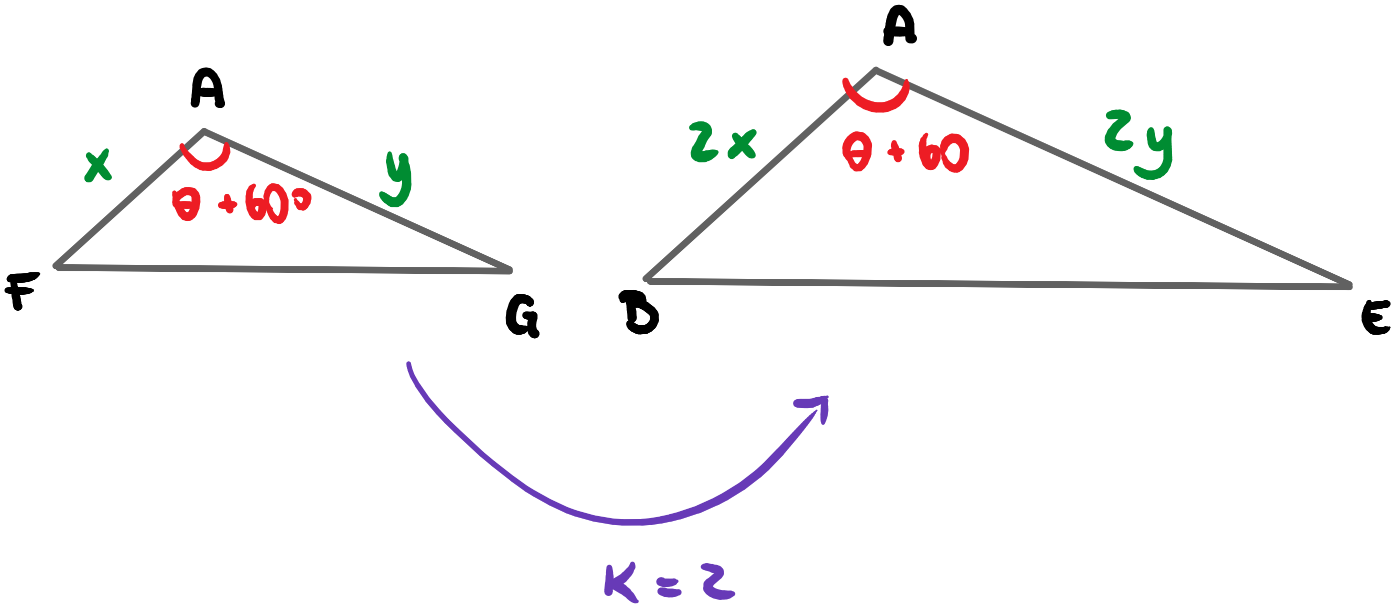

$$\boxed{\frac{FG}{BE} = \frac{1}{2}}$$

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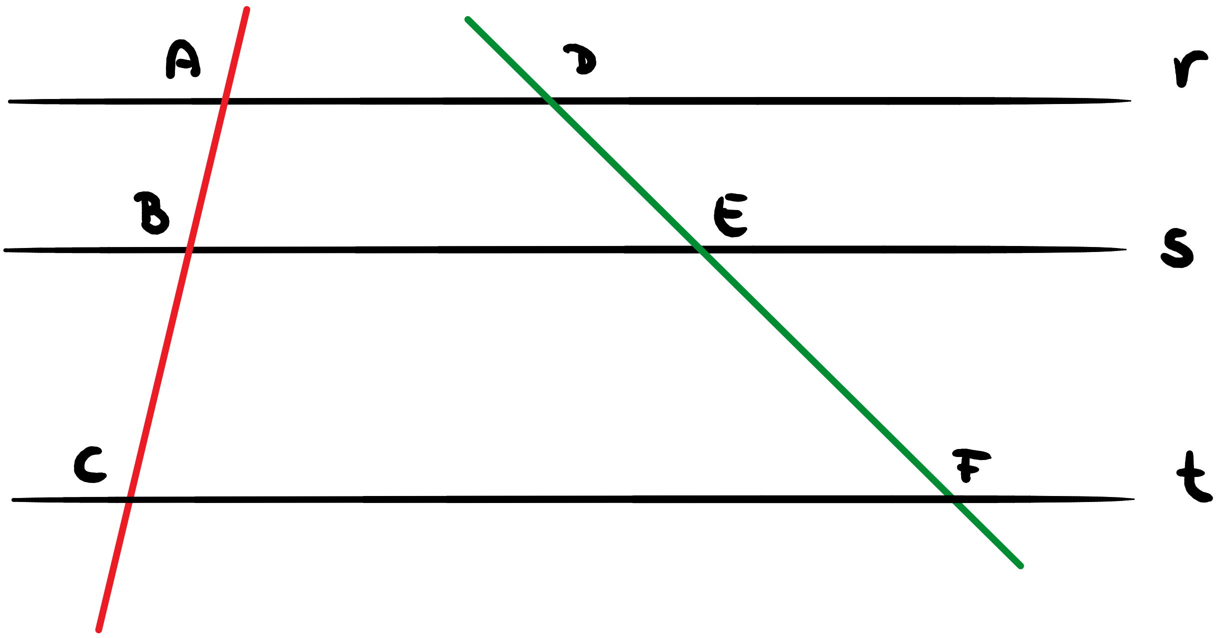
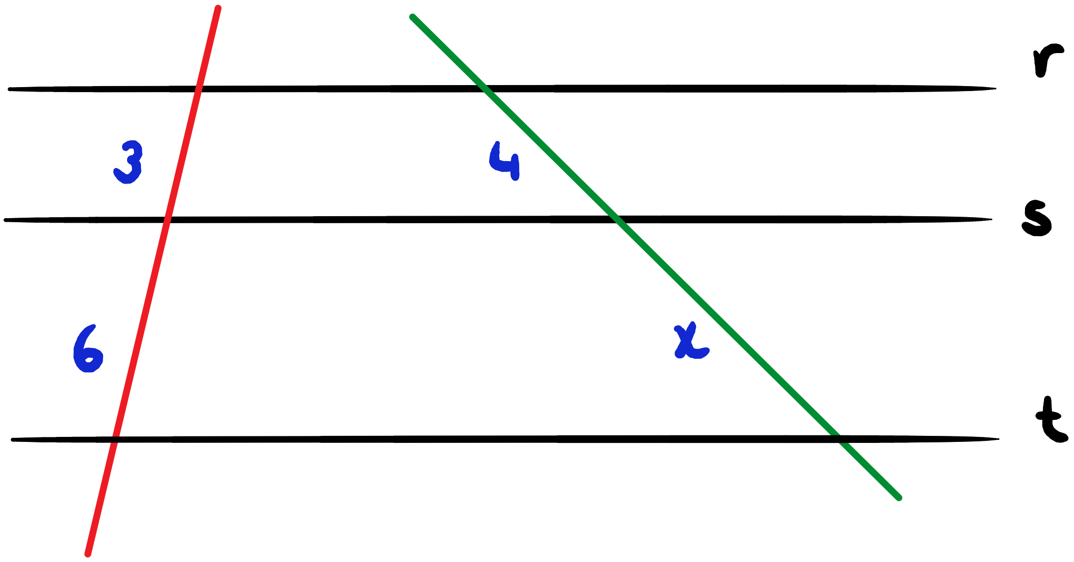
TEOREMA DE TALES

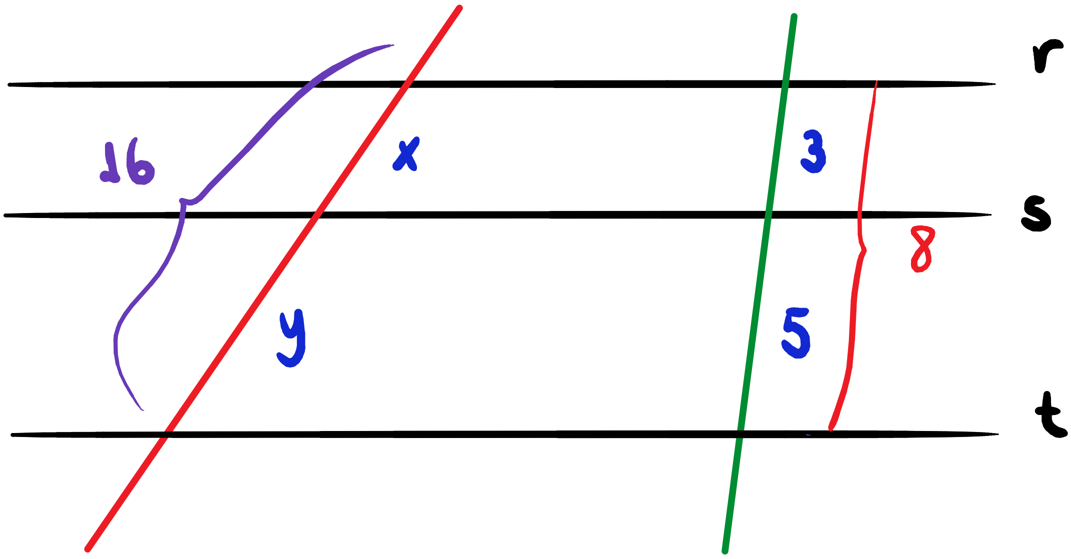
SEGMENTOS DE RETA ENTRE PARALELAS SÃOPROPORCIONAIS!



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



$$\frac{x}{\cancel{6}_{2}} = \frac{4}{\cancel{3}_{1}} \rightarrow \underline{x = 8}$$

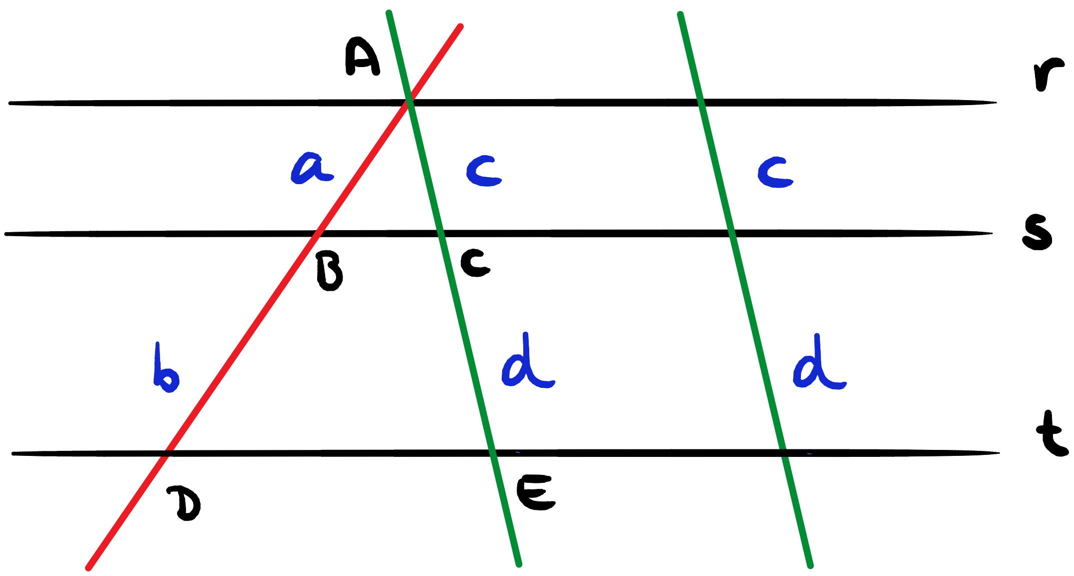


$$\frac{x}{3} = \frac{\cancel{16}^{2}}{\cancel{8}_{1}} \qquad\qquad \frac{y}{5} = \frac{16}{8}$$

$$x = 6 \qquad\qquad y = 10$$

TEOREMA DE TALES

DEMONSTRAÇÃO



$$\triangle ABC \sim \triangle ADE$$

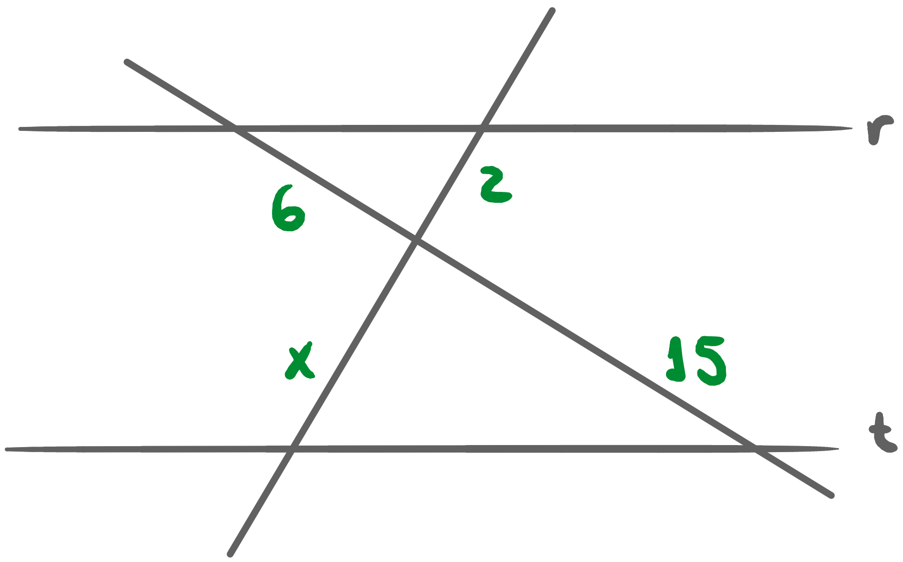
$$\frac{a+b}{a} = \frac{c+d}{c} \qquad \frac{b}{a} = \frac{d}{c}$$

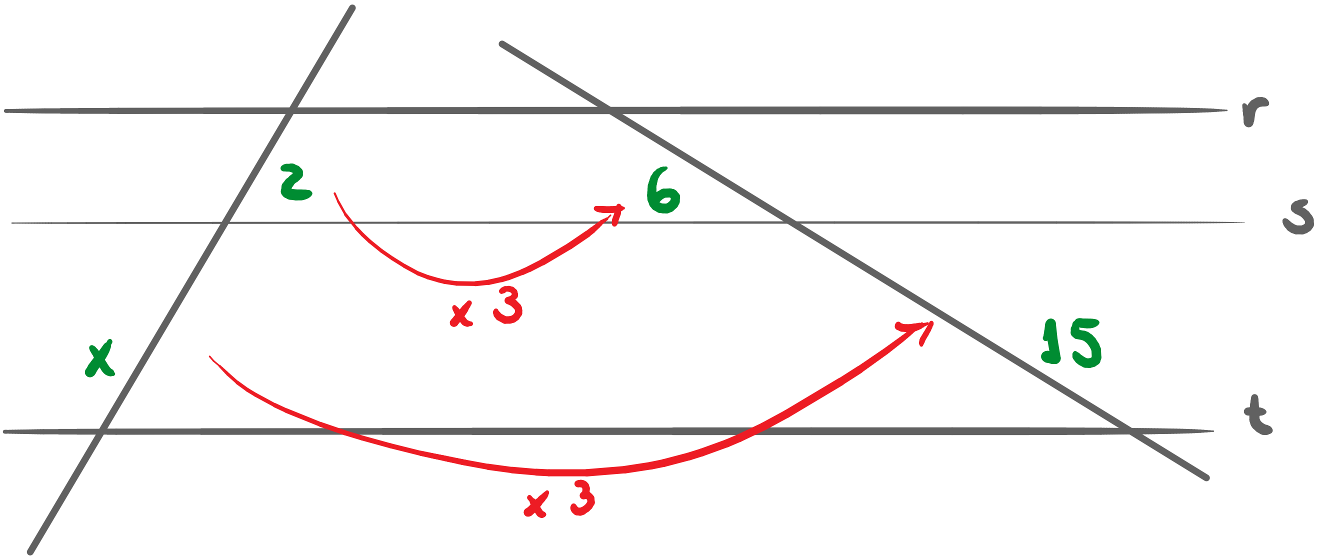
$$\cancel{\frac{a}{a}} + \frac{b}{a} = \cancel{\frac{c}{c}} + \frac{d}{c} \qquad a \cdot d = b \cdot c$$

$$\boxed{\frac{a}{c} = \frac{b}{d}}$$

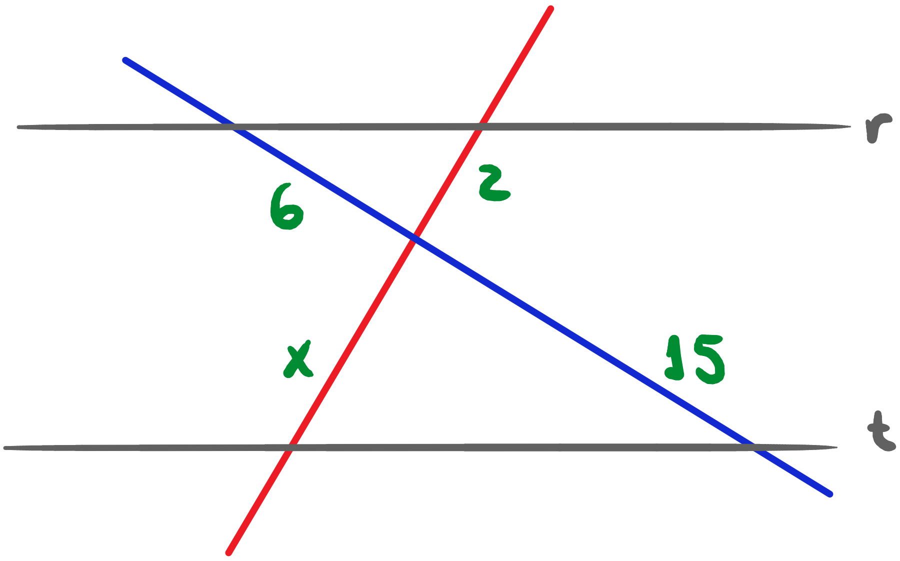
EXEMPLO

CALCULE O VALOR DE x.





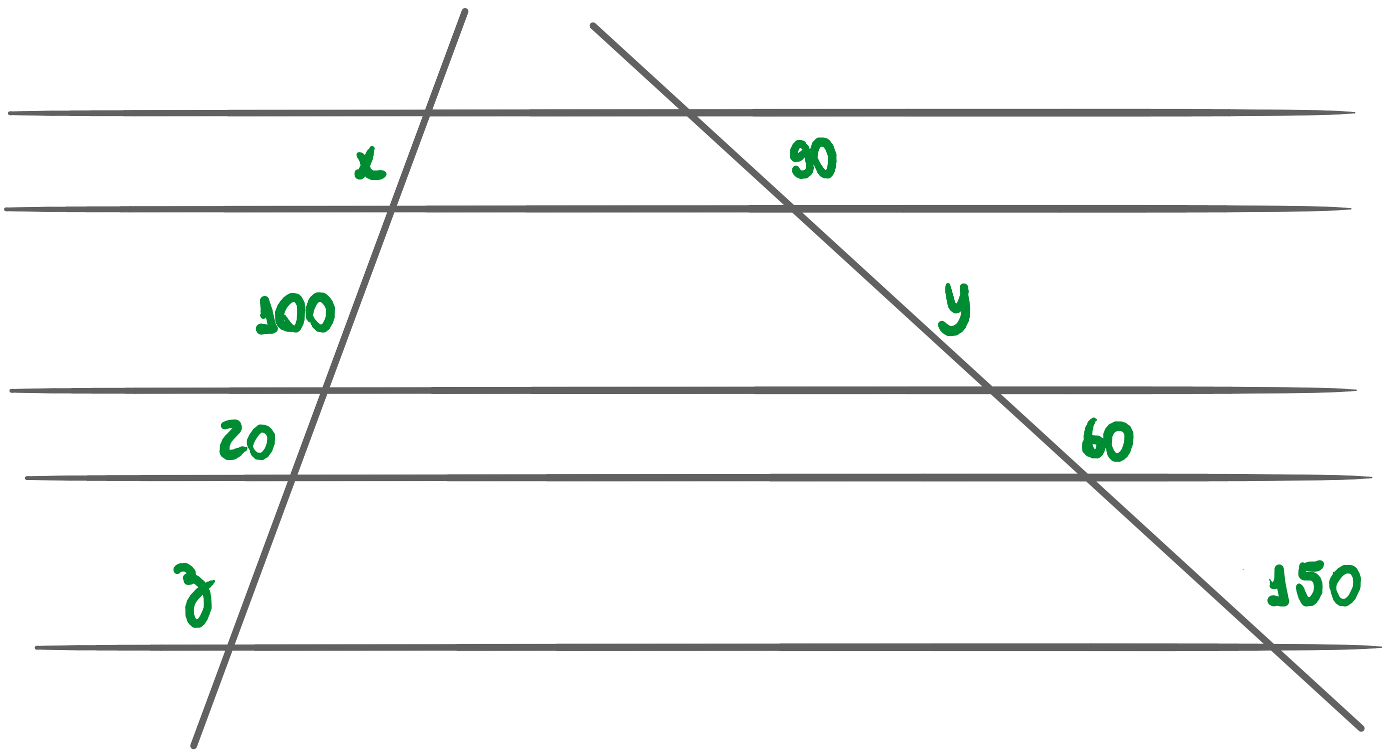
$$\frac{x}{\cancel{15}_{\,5}} = \frac{\cancel{2}\ 1}{\cancel{6}\ \cancel{3}\ 1} \longrightarrow x = 5$$

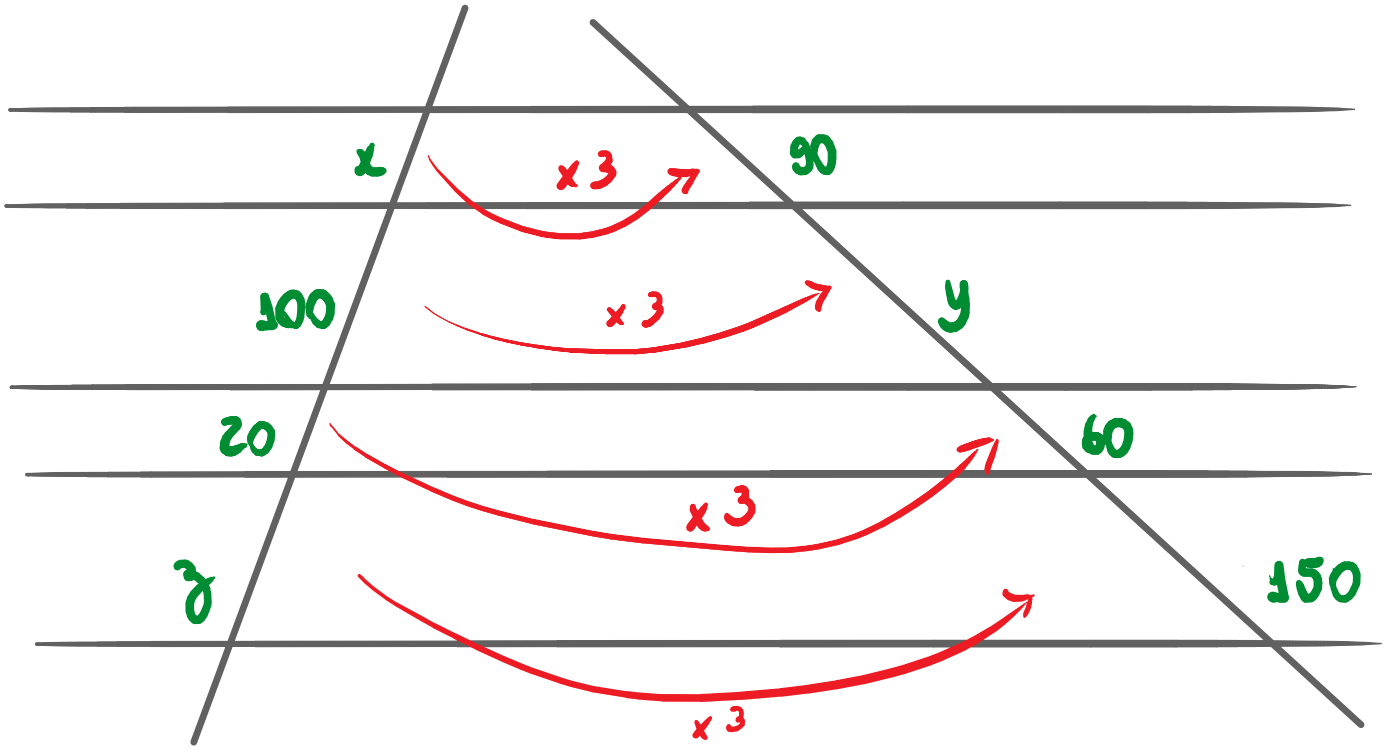


$$\frac{2}{6} = \frac{x}{15} \longrightarrow \underline{x = 5}$$

EXEMPLO

CALCULE OS VALORES DE x, y, z.





$$3 \cdot x = 90 \longrightarrow x = 30$$

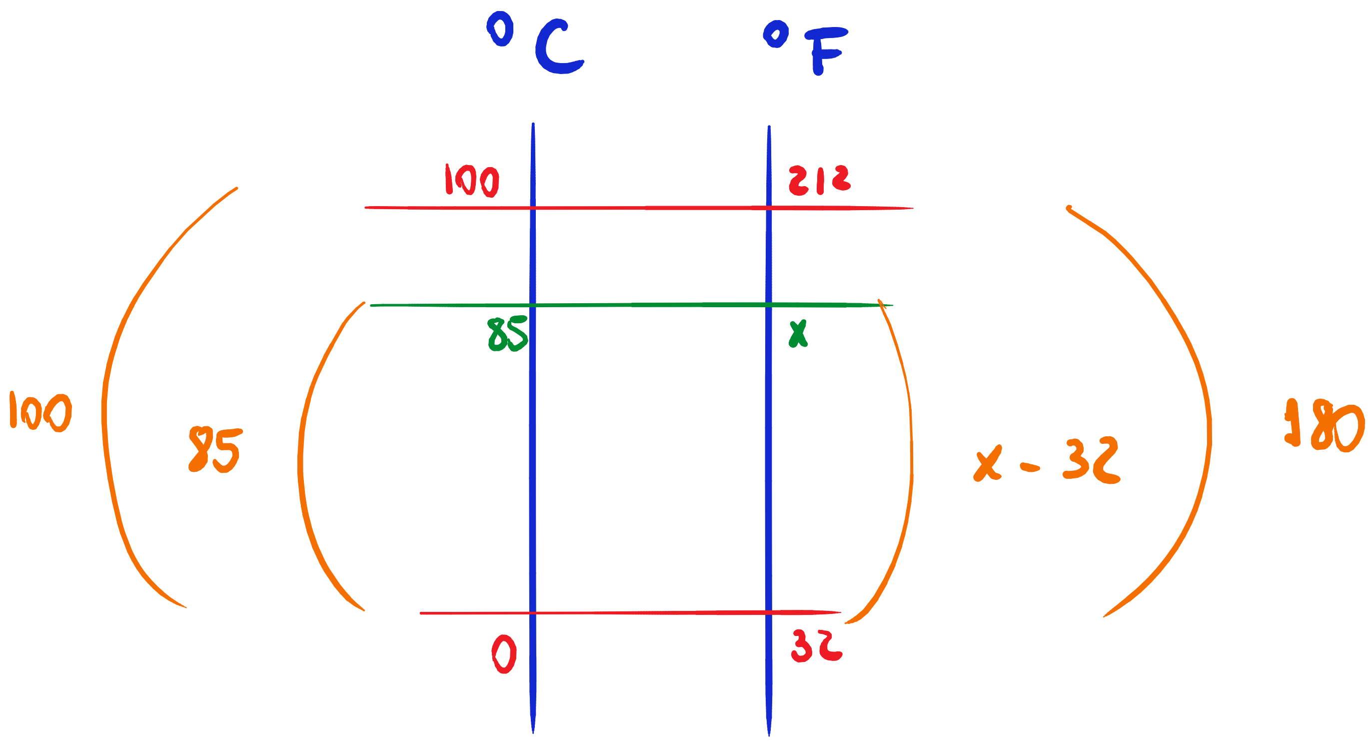
$$100 \cdot 3 = y \longrightarrow y = 300$$

$$3 \cdot z = 150 \longrightarrow z = 5$$

EXEMPLO

COMPARANDO AS ESCALAS DE TEMPERATURA CELSIUS E FAHRENHEIT, SABE-SE QUE 0°C = 32°F E 100°C = 212°F.

DESSA MANEIRA, CALCULE QUAL TEMPERATURA EM FAHRENHEIT CORRESPONDE A 85°C.



$$\frac{x - 32}{\cancel{85}_{17}} = \frac{\cancel{180}^{\,9}}{\cancel{100}_{\cancel{5}_{1}}}$$

$$x - 32 = 153$$

$$\underline{x = 185\,°F}$$